MODELING AND ANALYSIS OF A CLOSED-LOOP SUPPLY CHAIN FOR ALUMINUM ENGINE MANUFACTURING

By

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ABSTRACT

The closed-loop supply chain of an aluminum engine manufacturing operation is investigated. The push for the recycling of aluminum components in auto industry is motivated by profit incentives (i.e., the recovery of a valuable element) as well as by legal obligations (i.e., to comply with the requirements of the Extended Product Responsibility legislation), which have, therefore, given rise to the development of closed-loop supply chain.

The proposed research presents a planning model for the closed-loop process that includes purchasing, production, and end-of-life product collection and recycling/remanufacturing in the context of an aluminum engine manufacturing and recycling operation.

The model is a multi-echelon general integer linear program with the objective of minimizing the total costs in the network subject to structural and functional constraints. The model may be employed to make decisions regarding raw material procurement, production, recycling and inventory levels, and the transportation activities in the network.

DEDICATION

To my grandmothers

and

my parents

ACKNOWLEDGMENTS

I would like to take this opportunity to express my special and deepest gratitude to my supervisor, Dr. Reza S. Lashkari for his encouragement, support and fruitful guidance throughout my study.

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CHAPTER 1 INTRODUCTION

In the last decade or so, the concept of Supply Chain Management (SCM) has been adopted as a manufacturing paradigm for improving the competitiveness of an enterprise. In order to improve responsiveness and flexibility of manufacturing organizations, the SCM is considered as a competitive strategy for integrating suppliers and customers (Gunasekaran, 2004).

Supply chain management is a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and retailers, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize systemwide costs while satisfying service level requirement (Simchi-Levi, et al., 2000).

A supply chain is a network of facilities and distribution options that performs the function of procurement of materials, transformation of these materials into intermediate and finished products and the distribution of these products to customers (Ganeshan and Harrison, 1995).

A supply chain not only includes the manufacturer and suppliers, but also transporters, warehouses, retailers, and customers themselves. It consists of all the parties directly or indirectly involved in fulfilling a customer request (Chopra and Meindl, 2003).

1.1 Forward supply chain

The forward supply chain, which is also known as the forward logistics network (Figure 1.1), consists of suppliers, manufacturers, distribution centers, and retailers outlets, as well as raw materials, work-in-process inventory and finished products that flow between the facilities (Simchi-levi, et al., 2000).

Because different facilities in the supply chain may have diverse, conflicting, objectives and the supply chain is a dynamic system that changes over time, it is difficult to integrate the supply chain.

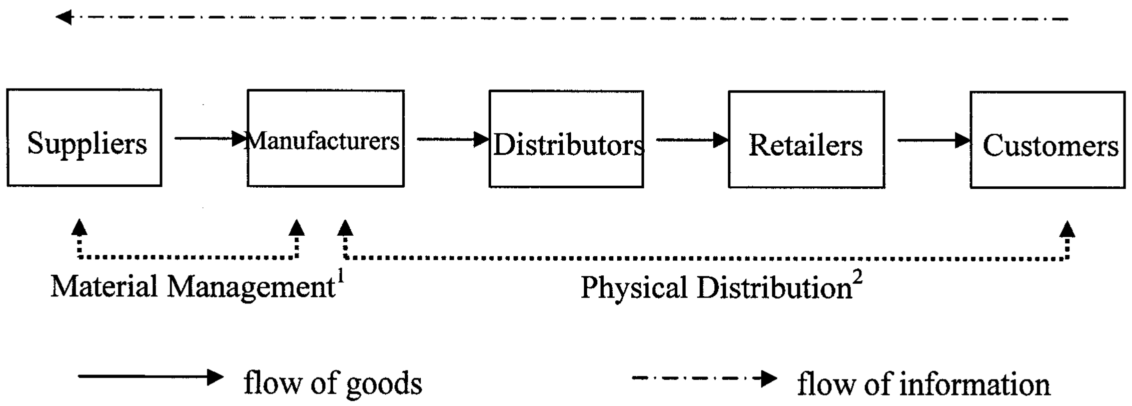

Fig 1.1 Forward supply chain (revised from Min and Zhou, 2002)

1.2 Reverse supply chain

The input sources of the reverse supply chain are usually the end-of-life returns, commercial returns and warranty returns, and the destinations may be venders, dealers, reprocess operations (remanufacture, refurbish, reuse), charities, recyclers and landfills.

The activities creating a continuous process to deal with returned products until they are appropriately recovered or disposed of, are collection, cleaning, disassembly, testing and sorting, storage, transport, and recovery operations in reverse logistics system. The recovery operations can be represented as one or a combination of several main recovery options, such as reuse, repair, refurbishing, remanufacturing, cannibalization and recycling.

A reverse supply chain is shown in Figure 1.2.

1.3 Closed-loop supply chain

Regarding the end-of-life issues, profit-oriented motivations and legal motivations force companies to integrate this product life phase into existing supply chain, forming a

[1] Material Management: including purchase and storage of raw materials, parts and supplies.

[2] Physical Distribution: all outbound logistics activities including order receipt and processing, inventory deployment, storage and handling, outbound transportation, consolidation, pricing, promotional support related to providing customer service

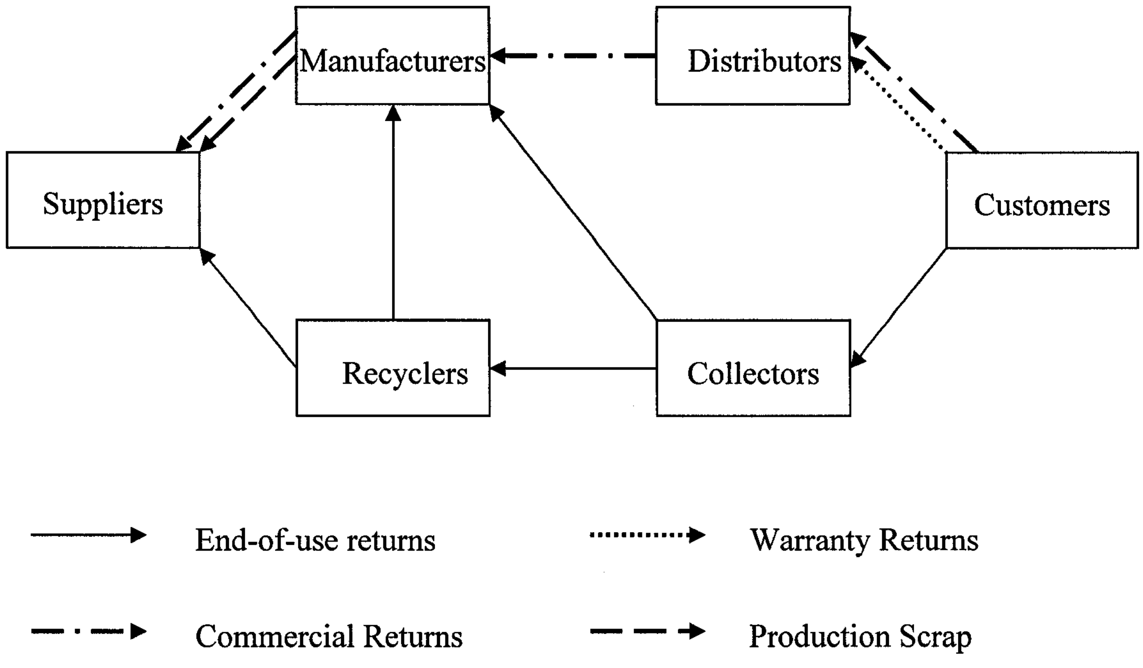

Fig 1.2 Reverse supply chain (revised from Fleischmann, 2001)

closed-loop supply chain, which combines the forward supply chain and the reverse supply chain (Schultmann, et al., 2006).

Besides the traditional forward processes for forward movement of goods to the consumer, the closed-loop supply chain having a number of activities required for reverse supply chain is designed to consider the purchase and return flows of products (Guide, et al., 2003). Figure 1.3 shows a closed-loop supply chain.

1.4 The forward logistics and reverse logistics, and their differences and relations

The Council of Logistics Management has defined the Logistics as "the process of planning, implementing, and controlling the efficient, effective flow and storage of goods, services and related information from the point of origin to the point of consumption for the purpose of conforming to customer requirements." (Bowersox and Closs, 1996)

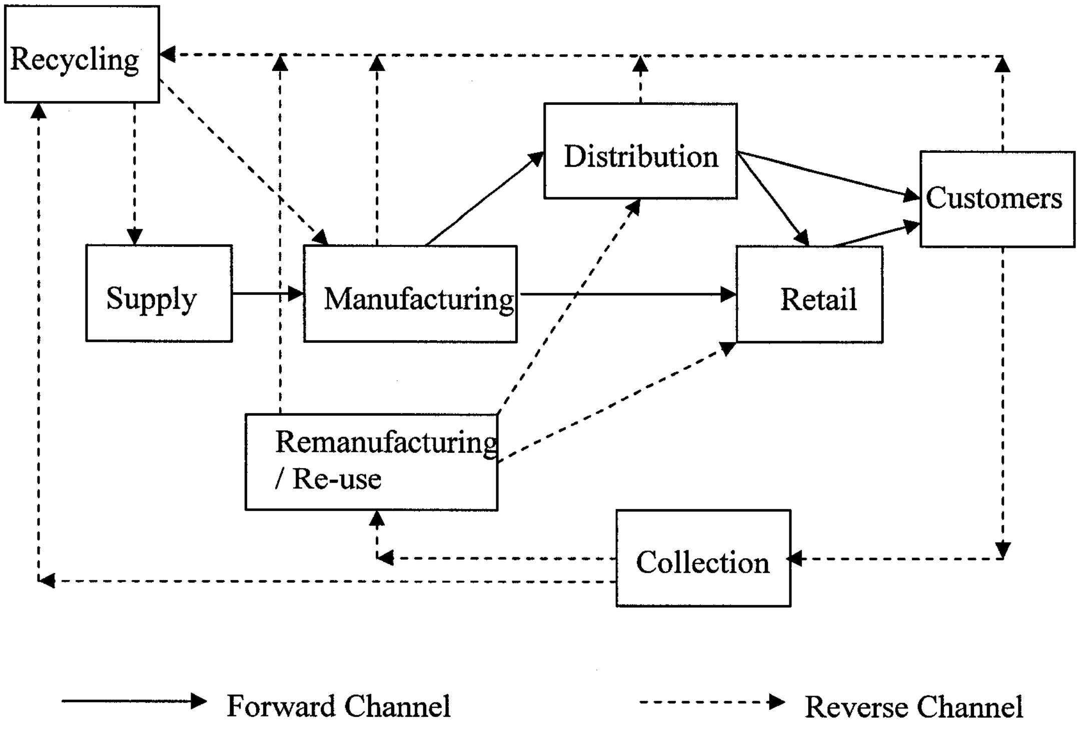

Fig 1.3 Closed-loop supply chain (revised from Kumar and Malegeant, 2006)

The Reverse Logistics Executive Council has defined the Reverse Logistics as "The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal."[3]

Fleischmann, et al. (1997) indicated that reverse logistics is "not necessarily a symmetric picture of forward distribution," and that it is much more reactive and much less visible compared with the forward logistics flow.

The differences between the forward and reverse logistics include a wide variety of viewpoints. For example, in the reverse logistics, there is lack of uniformity in the physical condition of products, while there is no sorting and evaluation of product in the forward logistics. The reverse logistics offers many challenges and opportunities which do not exist in the forward logistics.

[3] Information is available at http://www.rlec.org/glossary.htm

Table 1.1 shows a comparison of various features of forward and reverse logistics, and Table 1.2 shows a cost comparison between forward logistics and reverse logistics.

Category	Forward Logistics	Reverse Logistics
Forecasting	*Straightforward*	*Difficult*
Destination	*Many destinations*	*One destination*
Product Quality	*Uniform*	*Different*
Product Packaging	*Uniform*	*Often Damaged*
Destination/Routing	*Clear*	*Unclear*
Disposition Options	*Clear*	*Unclear*
Pricing	*Uniform*	*Different*
Arrive On-time	*Important*	*Not considered as priority*
Inventory management	*Consistent*	*Not consistent*
Handling	*Uniform*	*Complex*
Supply	*Consistent*	*Uncertainty*

Table 1.1 Differences between forward logistics and reverse logistics (modified from Tibben-Lembke and Rogers, 2002)

Cost	Forward Logistics	Reverse Logistics
Sorting/ Collecting	*Does not exist*	*Important*
Quality diagnosis	*Lower*	*Higher*
Handling	*Lower*	*Higher*
Holding	*Lower*	*Higher*
Reprocessing	*Does not exist*	*Important*

Table 1.2 Cost comparisons between forward logistics and reverse logistics (modified from Tibben-Lembke and Rogers, 2002)

1.5 Outline of the proposed research

In this research, we propose a closed-loop supply chain model of aluminum engine production, recycling and remanufacturing.

The proposed model is based on the operations of Ford Motor Company in the United States. Ford Motor Company, one of the largest automotive manufacturers in the world based in Dearborn, Michigan, manufactures and distributes automobiles in 200 markets. The research is centered around 5 Ford engine plants in the United States.

1.6 Motivations of the proposed research

Many papers have focused on reverse logistics, but research work on the planning and optimization of reverse logistics network design is limited.

Legal obligations as well as profit incentives to recover value in recycled products have given rise to the need for efficient supply chain designs.

The resources used by industry are limited, and energy is saved when recycled material is used rather than new material, so an efficient closed-loop supply chain is clearly needed.

Using recycled aluminum saves 95 percent of the energy required to make new aluminum, and approximately 60% to 70% of aluminum used in today's vehicles is sourced from recycled metal[4]. About 85 to 90% of post-consumer automotive aluminum scrap, at least on billion pounds per year, is recycled today[4]. Remanufactured engines could be produced with 68% to 83% less energy and it pays to note that 70% of the cost to build new engines lies in the materials, while only 30% lies in the labor (Smith and Keoleian, 2004). Close to 2.2 million engines are remanufactured annually by the North America engine remanufacturing industry and nearly 6,000 machine shops in North America remanufacture engine[5]. Approximately 17 truckloads worth of used diesel engines and other parts are dumped at a receiving facility every day in the U.S. (Hindo, 2006). The usage of aluminum in vehicles will be increasing — worldwide — from 3% to 3.5% per year for the balance of this decade[6].

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[4] Information is available at http://www.aluminum.org/Content/NavigationMenu/The_Industry/Transportation_Market/Auto_Truck/RecyclabilityAndScrapValue.htm
[5] Information is available at http://www.pera.org/index.htm
[6] Information is available at http://www.aluminum.org/ANTemplate.cfm?IssueDate=09/01/2006&Template=/ContentManagement/ContentDisplay.cfm&ContentID=10387

CHAPTER 2 LITERATURE REVIEW

The area of reverse logistics is currently drawing great interest both commercially and academically. Reverse logistics aims at improving the utilization of used products, or their parts, through recycling, remanufacturing or other forms of recovery (Ayres, 1995).

Legal obligations as well as profit incentives to recover value in returned products have given rise to the need for efficient supply chain designs. Product returns are a relevant issue for many industries, for instance, carpets, computers, printers, automobiles, etc.

2.1 Reverse logistics

Reverse distribution is the collection and transportation of used products, which may occur through the original forward channel, through a separate reverse channel, or through combinations of the forward and the reverse channels.

Krumwiede and Sheu (2002) investigated current industry practices in reverse logistics business, and employed a decision-making model to guide the process of examining the feasibility of implementing reverse logistics in third-party providers such as transportation companies. The model provided assistance to third-party logistic companies in making the decision whether or not to enter the reverse logistics business.

Jayaraman et al. (2003) discussed the reverse distribution network of an electronic equipment remanufacturing company in the US and developed a decision model to minimize reverse distribution costs. The model consisted of a single-source plant with a restricted number of collection sites and refurbishing sites that could be opened. The authors used a heuristic solution methodology to solve the model.

Nagurney and Toyasaki (2005) developed a multi-tiered e-cycling network model consisting of four tiers of nodes (sources of the electronic waste, recyclers of the electronic waste, processors of the electronic waste and demand market) for a reverse supply chain. They obtained the decision-makers' optimality conditions and provided the governing equilibrium conditions, in conjunction with the variational inequality formulation. The solution generates the material flow and the price. The authors also gave some directions for future research. One is to extend the model to a reverse supply chain model with random supply of electronic wastes associated with the sources. Another extension is to integrate the production and the distribution systems of electronic products.

Schultmann et al. (2006) have modeled the reverse logistics aspects of the end-of-life vehicle (ELV) treatment in Germany using vehicle routing planning. The authors proposed a model in which the objective function minimizes the total length of all the tours necessary for the vehicle routing problem (VRP). They also indicated that flexible algorithms are necessary to compare different scenarios of establishing a reverse supply chain for collecting secondary material.

Lieckens and Vandaele (2007) have extended traditional facility location-allocation models (formulated as mixed integer linear programs and determining which facilities to open that minimize the cost while supply, demand and capacity constraints are satisfied) by introducing queuing relationships into the network in order to incorporate a product's cycle time and inventory holding costs, in addition to dealing with the higher degree of uncertainty and congestion, typical characteristics of these networks. The mixed integer nonlinear program (MINLP-model) is presented for a single-product-single-level-location model of a reverse network. The authors used the differential evolution (DE) algorithm to solve the MINLP-model.

In the area of remanufacturing, Guide and Srivastava (1998) discussed the scheduling policies for remanufacturing shops based on the information from turbine jet engine remanufacturing. In the context of a remanufacturing environment, they examined the location of inventory buffers and their impact on other managerial operating decisions.

Mahadevan, et al. (2003) investigated an inventory system with manufacturing and remanufacturing and employed a "push" policy. They examined the operation of the system as a function of return rates, backorder costs, manufacturing and remanufacturing lead times. The authors tested their heuristics by means of simulation using PROMODEL.

Smith and Keoleian (2004) developed a life-cycle assessment model (LCA) to investigate the energy saving and pollution prevention that are achieved through remanufacturing an engine compared to an OEM manufacturing a new one. The model result showed a 68%~83% reduction in energy use and a 73%~87% reduction in carbon dioxide emissions. The results also indicated more than 50% reduction for each of remaining emissions. The consumption of raw material was reduced by 26%~90% and solid waste generation was reduced by 65%~88% as well.

2.2 Closed-loop supply chain

Spengler (2003) presented the design and implementation of a decision support system for electronic scrap recycling companies in Germany. The author developed a mixed-integer linear programming model which maximizes the total achievable marginal income subject to mass balance equations and capacity restrictions for the recycling process of dismantling and bulk recycling of discarded products. The model was solved using LINGO.

Beamon and Fernandes (2004) presented a closed-loop supply chain in which manufacturers produce new products and remanufacture used products. They made decisions about which warehouses and collection centers should be open, and which warehouses should have sorting capabilities and how much material should be transported between each pair of sites using the multi-period mixed-integer programming model. There are four echelons including manufacturers, warehouses, customer zones and collection centers in their model.

Sheu, et al. (2005) proposed a linear multi-objective programming model to deal with integrated logistics operational problems of Green-supply chain management (G-SCM). The model was developed to optimize the operations of both the integrated logistics and the corresponding used-product reverse logistics in a given chain of five layers based on a real world case study for a computer manufacturer.

Zhang and Lashkari (2005) investigated a closed-loop supply chain of a lead-acid battery manufacturing process. The model is a multi-objective (minimizing the total cost and minimizing the pollution emission), multi-echelon mixed integer linear program. The results may be used to make decisions regarding raw material procurement, production, recycling and inventory levels, and the transportation modes between the echelons.

Vlachos, et al. (2007) presented a system dynamics model for dynamic capacity planning of remanufacturing in closed-loop supply chains. They focus on a single product closed-loop supply chain in which the forward chain has two echelons: producer and distributor. The reuse activity is remanufacturing. They dealt with the development of efficient capacity planning policies for remanufacturing facilities in reverse supply chains, taking into account not only economic but also environmental issues.

Ko and Evans (2007) presented a mixed integer nonlinear programming model for dynamic supply chain management by third party logistics providers (3PLs), which belongs to a class of multi-period, two-echelon, multi-commodity, capacitated location models. This model has the objective of minimizing the total costs incurred in the forward and reverses flows. Since such network design problems belong to a class of NP hard problems, a genetic algorithm-based heuristic is proposed to solve it.

Listeş (2007) presented a generic two-stage (plant and market) stochastic programming model for the design of closed loop networks and used a decomposition-based approach to solve this problem. The author considered only a single planning period in the proposed model. The model explained a number of alternative scenarios which may be constructed based on critical levels of design parameters such as demand and returns.

Lu and Bostel (2007) develop an algorithm to solve the two-level location model with three types of facility which are producers, remanufacturing centers and intermediate to be sited, considering “forward” and “reverse” flows that cover remanufacturing activities. For further development, the authors also point out that this research can be extended to facility location problems with capacity as well as to other types of reverse logistics systems. The objective of the location model is to minimize the costs of setting up facilities, shipping and receiving products.

CHAPTER 3 SYSTEM DESCRIPTION

3.1 The life cycle of aluminum engines

The aluminum engine manufacturing, recycling, and remanufacturing under consideration in this research is depicted in Figure 3.1.

The aluminum casting plant purchases aluminum ingots and recycled aluminum, and mixes them to make new aluminum cylinder blocks and cylinder heads, which are then delivered to the engine plants according to a schedule.

The engine plant produces engines depending on customer demand. It purchases other aluminum engine parts from outside suppliers. The engine plant maintains safety stocks for both the components and the assembled engines. The safety stock is normally equal to a two-day production level. The plant also sends new engines to dealerships for replacing warranty returns.

The assembled engines are delivered to auto assembly plants to be installed in vehicles. After assembly, vehicles are sent to dealerships. Customers purchase vehicles from the dealers. Dealership sends warranty replacements from customers to collection centers, and the dealership orders new engines from engine plants to replace the warranty returns.

The end-of-life vehicles are returned to collection centers. Nowadays, a collection center uses an elaborate electronic system to help in deciding what to dismantle. If the engine can be rebuilt then the collection center takes the engine out and sends it to a remanufacturing center. If not suitable for remanufacturing, the engine will be left in the vehicle which will be flattened.

Flattened hulks are shipped to shredders which pulverize them into fist-sized pieces in minutes. Valuable ferrous and non-ferrous metals are removed magnetically using complex floatation systems, and the shredder "fluff" is sent to landfills.

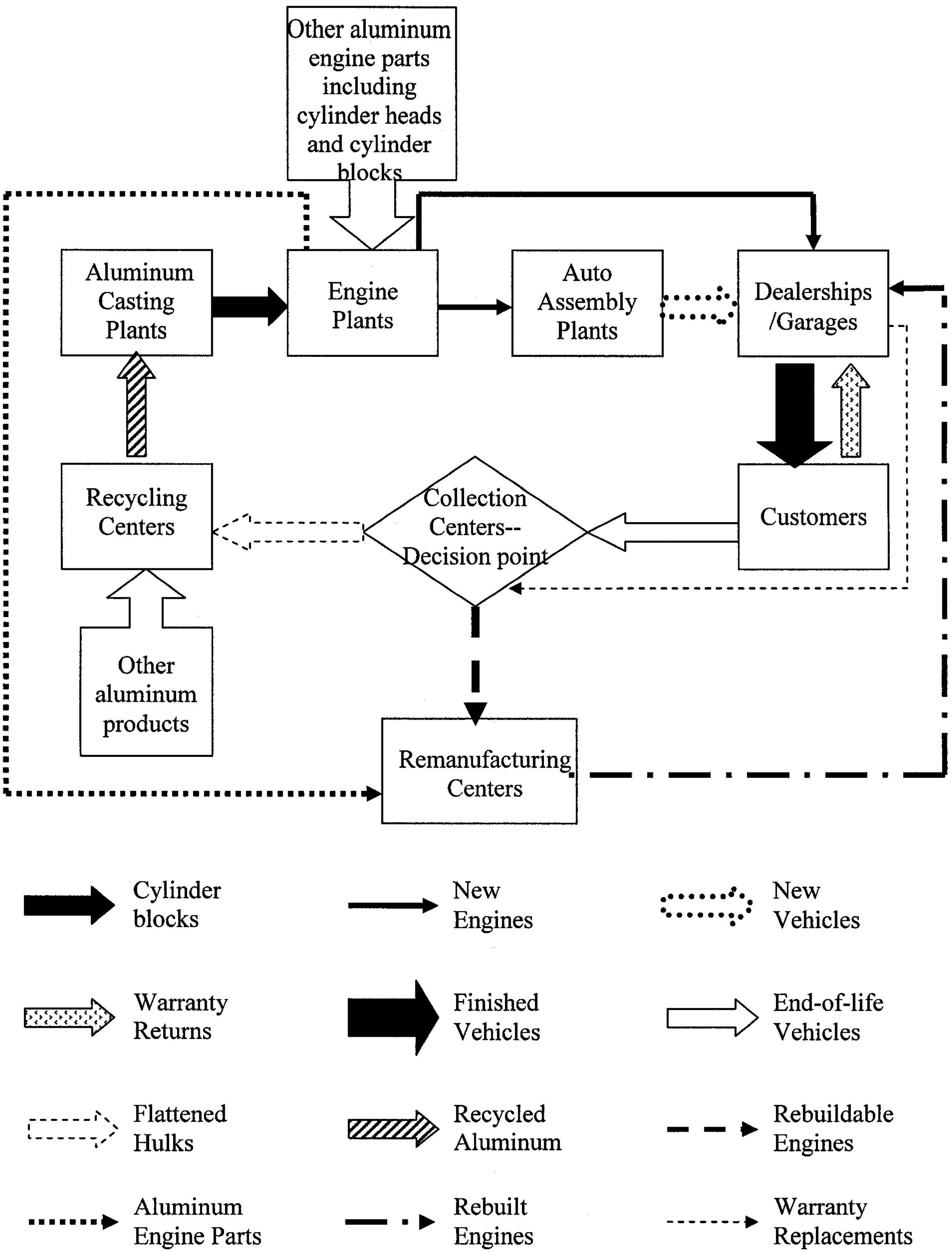

Figure 3.1 A closed-loop supply chain for aluminum engines

3.2 Problem statement

3.2.1 Components of the proposed model

The closed-loop model includes:

- 1 aluminum casting plant
- 5 engine plants
- 13 auto assembly plants
- 3 collection centers
- 2 recycling centers
- 2 remanufacturing centers

Table 3.1 presents the system under consideration. The engines are divided into three product families, aluminum-aluminum engine (i.e., aluminum cylinder head and aluminum cylinder block), aluminum-iron engine (i.e., aluminum cylinder head and iron cylinder block) and iron-iron engine (i.e., iron cylinder head and iron cylinder block). In the proposed research, we only consider the first two families. There are 5 Ford engine plants in the U.S., which produce 7 engine types within 2 product families. These are then sent to 13 auto assembly plants. For example, engine plant 1 produces engine types 1 and 2 belonging to product family 1. Engine type 1 is sent to auto assembly plant 12, and engine type 2 is sent to auto assembly plant 8. Mazda 6 and Mustang are assembled at auto assembly plant 1. Auto assembly plants 9, 10, and 11 are not within the United States.

Ford no longer owns an aluminum casting plant, and there are only two aluminum casting plants in North America that are part of the Nemak joint venture (see Table 3.2), all operating in Canada. One plant sends its products to some of the 5 engine plants in the U.S. The aluminum casting plant sends engine parts for engine type 3 to engine plant 2, for engine type 4 to engine plants 2 and 3, for engine type 6 to engine plant 5. The rest of the aluminum engine parts are purchased by Ford from outside suppliers. These suppliers do not belong to Ford, and therefore are not included in our study.

Although auto assembly plants 9, 10, and 11 and the aluminum casting plant are not within the United States, their costs are included in this study because the operation of the 5 engine plants are within the United States.

Although there are many collection, recycling and remanufacturing centers within the United States, we only use 3 collection centers, 2 recycling centers and 2 remanufacturing centers to represent the entire group. While they do not belong to Ford, the closed-loop supply chain must be established in order to support the end-of-life vehicle returns. From another perspective, the costs in the reverse channel also have an effect on the forward channel.

Engine plant, *m*	**Product family, *q***	**Engine type, *n***	**Auto assembly plants, *a***
1	**1** (Aluminum cylinder head and aluminum cylinder block engine)	**1.** 2.0L I4	12. Ford Focus
		2. 2.3L I4	8. Ford Ranger
2		**3.** 2.5L V6	10. Jaguar X-Type, Mondeo
		4. 3.0L V6	1. Mazda 6 4. Escape 9. Ford Fusion, Mercury Milan, Lincoln Zephyr 10. Jaguar X-Type, Jaguar S-Type, Jaguar XJ-Type, Mondeo
3		**4.** 3.0L V6	2. Ford Freestyle, Mercury Montego Ford Five Hundred
4		**5.** 3.5 L V6	11. Edge, MKX
5		**6.** 4.6L V8	1. Mustang
5	**2** (Aluminum cylinder head and iron cylinder block engine)	**6.** 4.6L V8	3. F-150 4. F-150 5. F-150 6. Ford Explorer, Ford Explorer Sport Trac, Mercury Mountaineer 7. Ford E-Series 13. Lincoln Town Car
		7. 5.4L V8	1. Mustang

Table 3.1 Products produced at engine plants

Aluminum casting plant, *s*	Aluminum engine parts, *i*	Parts for engine type, *n*
1	1. Aluminum Cylinder Blocks	3. 2.5L V6 4. 3.0L V6 6. 4.6L V8
Outside Suppliers (not a part of Ford operations)	2. Other Aluminum Cylinder Heads and Blocks	Other Cylinder Heads Other Cylinder Blocks
Outside Suppliers (not a part of Ford operations)	3. Other Aluminum Parts	Oil Pumps, Water Pumps, Pistons, Rocker Arm Covers, Front Covers

Table 3.2 Aluminum engine parts provided by suppliers

3.2.2 Characteristics of the proposed model

The main characteristics of the proposed model are as follows:

- closed-loop supply chain consideration
- different transportation lead times at different locations in the chain
- both the virgin raw material and the recycled materials will be considered as incoming material flow into the casting operations
- deterministic demand for finish goods and end-of-life products recycled

CHAPTER 4 MATHEMATICAL MODEL

4.1 Mathematical modeling

4.1.1 Assumptions

The following assumptions have been considered in the model's formulation:

1. All in-transit inventory transportation costs are accounted for at the source. The in-transit shipments from engine plants to auto assembly plants, for example, are charged to engine plants they originated from. The in-transit inventory in three of these stages — from engine plants to remanufacturing centers, from collection centers to remanufacturing centers and from remanufacturing centers to dealerships — is small enough in number to be insignificant to our study.
2. All transportation cost are accounted for at the source too. The transportation costs from engine plants to auto assembly plants, for example, are charged to engine plants.
3. The number of returned end-of-life vehicles is calculated based on the number of vehicles that are retired every year multiplied by Ford's market share.
4. The rate of material loss is 0.10 during the manufacturing at the aluminum casting plant.
5. The remanufacturing centers do not keep inventories of engine parts i=3 for rebuilding engines.
6. No inventories are held at the collection center, since these centers act as decision points in the chain.
7. At the engine plants, the labor requirements are met through regular-time, overtime, additional labor hiring and layoffs. At the aluminum casting plant and at the remanufacturing centers the labor requirements are met through regular time and overtime. A worker operates 8 hours a day, 5 days a week.
8. The labor costs accounted for in the model are related to engine production, which includes the labor costs at the aluminum casting plants, engine plants, and the remanufacturing centers. The relevant labor costs at the auto assembly plants and the collection centers are included in the handling costs.

9. New engines sent from the engine plants to dealerships to replace warranty returns, engine warranty returns sent from customers to dealerships, and engine warranty replacements sent from dealerships to collection centers are small enough in numbers to be insignificant in our study. Their values are therefore assumed to be equal to zero.

4.1.2 Notations

$a \in A$ Index for auto assembly plants, where:

a=1 for engine type n=6, product family q=1;

a=1 for engine type n=7, product family q=2;

a=1, 4, 9, 10 for engine type n=4, product family q=1;

a=2 for engine type n=4, product family q=1;

a=3, 4, 5, 6, 7, 13 for engine type n =6, product family q=2;

a=8 for engine type n=2, product family q=1;

a=10 for engine type n=3, product family q=1;

a=11 for engine type n=5, product family q=1;

a=12 for engine type n=1, product family q=1;

$c \in C$ Index for collection centers, c=1,…,C;

$d \in D$ Index for dealerships, d=1,…,D;

$i \in I$ Index for aluminum engine parts; i=1, when s=1; and engine parts i=2, 3 are purchased from outside suppliers;

$k \in K$ Index for transport modes, k=1 means trucks, and k=2 means trains

$m \in M$ Index for the number of engine plants, where:

m=1 for engine types n=1, 2, product family q=1;

m=2 for engine types n=3, 4, product family q=1;

m=3 for engine type n=4, product family q=1;

m=4 for engine type n=5, product family q=1;

m=5 for engine type n=6, product family q=1, and for engine types n=6, 7, product family q=2;

$n \in N$ Index for the engine types, where n=1, 2, 3, 4, 5, 6 when product family q=1; and n=6, 7 when product family q=2;

$q \in Q$ Index for the product families, q=1, 2;

$r \in R$ Index for recycling centers, r=1,…,R;

$s \in S$ Index for aluminum casting plant, s=1;

$t \in T$ Index for time periods (day), t=1,…,20;

$u \in U$ Index for remanufacturing centers, u=1,…,U.

4.2 Mathematical formulations

4.2.1 Model description

The objective function is to minimize the total cost which includes purchasing costs, transportation costs, inventory costs, in-transit inventory costs, labor costs and handling costs and minus the revenue from selling engine parts. These components are described below.

Purchasing Costs:

|purchasing aluminum ingots from suppliers and recycled aluminum from recycling centers by an aluminum casting plant|

+ *Transportation Costs:*

|transporting engine parts from the aluminum casting plant to engine plants| + |transporting engines from engine plants to auto assembly plants| + |transporting vehicles from auto assembly plants to dealerships| + |transporting flattened hulks from collection centers to recycling centers| + |transporting engine parts from engine plants to remanufacturing plants| + |transporting rebuildable engines from collection centers to remanufacturing plants| + |transporting rebuilt engines from remanufacturing plants to dealerships|

+ *Inventory Costs:*

|inventory holding cost of recycled aluminum, aluminum ingots and cylinder blocks at the aluminum casting plant| + |inventory holding cost of engine parts and assembled engines at engine plants| + |inventory holding cost of assembled engines and new vehicles at auto assembly plants| + |inventory holding cost of rebuildable engines and rebuilt engines at remanufacturing centers|

+ *In-transit Inventory Costs:*

|in-transit transportation of engine parts from the aluminum casting plant to engine plants| + |in-transit transportation of assembled engines from engine plants to auto assembly plants| +

|in-transit transportation of new vehicles from auto assembly plants to dealerships| + |in-transit transportation of flattened hulks from collection centers to recycling centers|

+ *Labor Costs:*

|labor cost at the aluminum casting plant| + |labor cost at engine plants| + |labor cost at remanufacturing centers|

+ *Handling Costs:*

|handling cost of new vehicles at auto assembly plants| + |handling cost of end-of-life vehicles collected from customers at collection centers|

- *Revenue at engine plants*

|selling the engine parts from engine plants to remanufacturing centers|

The following classes of constraints are in effect:

1. *Production capacity* at the aluminum casting plant, engine plants, auto assembly plants, collection centers, recycling centers and remanufacturing centers
2. *Storage capacity* at the aluminum casting plant, engine plants, auto assembly plants and remanufacturing centers
3. *Production labor hour* at the aluminum casting plant, engine plants and remanufacturing centers
4. *Transport carriers' capacity*
5. *Inventory capacity*
   a. *Inventory balance* at the aluminum casting plant, engine plants, auto assembly plants, and remanufacturing centers
   b. *In-transit inventory balance*:
      from the aluminum casting plant to engine plants;
      from engine plants to auto assembly plants;
      from auto assembly plants to dealerships;
      from collection centers to recycling centers
   c. *Safety stock* at the aluminum casting plant, engine plants, auto assembly plants and remanufacturing centers

4.2.2 The objective function

4.2.2.1 PURCHASING COSTS

Parameters:

- *P1:* Percentage of aluminum ingots in the total amount of aluminum purchased by the aluminum casting plant *s*
- *P2:* Percentage of recycled aluminum in the total amount of aluminum purchased by the aluminum casting plant *s*
- *CP1:* Per unit weight purchasing cost of aluminum ingots from the suppliers
- *CP2:* Per unit weight purchasing cost of recycled aluminum from recycling centers

Decision variables:

- XC_{st}: The amount of aluminum, in lbs, purchased by aluminum casting plant *s* in time period *t*

[1] At the aluminum casting plant *s*, and in each period *t*, the total purchasing cost of the aluminum input consists of the purchasing cost of aluminum ingots and the purchasing cost of recycled aluminum:

$$\sum_{s=1}^{S}\sum_{t=1}^{T}(P1 \times CP1 \times XC_{st} + P2 \times CP2 \times XC_{st})$$

4.2.2.2 TRANSPORTATION COSTS

Parameters:

- $CTR1_{smk}$: Per unit cost of transportation from aluminum casting plant *s* to engine plant *m* using transport mode *k* ($/Full Truck Load, or $/FTL)
- $CTR2_{mak}$: Per unit cost of transportation from engine plant *m* to auto assembly plant *a* using transport mode *k* ($/FTL)
- $CTR3_{adk}$: Per unit cost of transportation from auto assembly plant *a* to dealership *d* using transport mode *k* ($/FTL)
- $CTR4_{crk}$: Per unit cost of transportation from collection center *c* to recycling center *r* using transport mode *k* ($/FTL)
- $CTR5_{muk}$: Per unit cost of transportation from engine plant *m* to remanufacturing center *u* using transport mode *k* ($/FTL)

$CTR6_{cuk}$: Per unit cost of transportation from collection center *c* to remanufacturing center *u* using transport mode *k* ($/FTL)

$CTR7_{udk}$: Per unit cost of transportation from remanufacturing center *u* to dealership *d* using transport mode *k* ($/FTL)

Decision variables:

$XTR1_{smkt}$: Number of FTL shipments from aluminum casting plant *s* to engine plant m using transport mode *k* in time period *t*

$XTR2_{makt}$: Number of FTL shipments from engine plant *m* to auto assembly plant *a* using transport mode *k* in time period *t*

$XTR3_{adkt}$: Number of FTL shipments from auto assembly plant *a* to dealership *d* using transport mode *k* in time period *t*

$XTR4_{crkt}$: Number of FTL shipments from collection center *c* to recycling center *r* using transport mode *k* in time period *t*

$XTR5_{mukt}$: Number of FTL shipments from engine plant *m* to remanufacturing center *u* using transport mode *k* in time period *t*

$XTR6_{cukt}$: Number of FTL shipments from collection center *c* to remanufacturing center *u* using transport mode *k* in time period *t*

$XTR7_{udkt}$: Number of FTL shipments from remanufacturing center *u* to dealership *d* using transport mode *k* in time period *t*

[1] The transportation cost of the total number of FTL shipments from aluminum casting plant *s* to engine plant *m* using transport mode *k* in time period *t*:

$$\sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR1_{smk}\, XTR1_{smkt}$$

[2] The transportation cost of the total number of FTL shipments from engine plant *m* to auto assembly plant *a* using transport mode *k* in time period *t*:

$$\sum_{m=1}^{M}\sum_{a=1}^{A}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR2_{mak}\, XTR2_{makt}$$

[3] The transportation cost of the total number of FTL shipments from auto assembly plant *a* to dealership *d* using transport mode *k* in time period *t*:

$$\sum_{a=1}^{A}\sum_{d=1}^{D}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR3_{adk}\, XTR3_{adkt}$$

[4] The transportation cost of the total number of FTL shipments from collection center *c* to recycling center *r* using transport mode *k* in time period *t*:

$$\sum_{c=1}^{C}\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR4_{crk}\, XTR4_{crkt}$$

[5] The transportation cost of the total number of FTL shipments from engine plant *m* to remanufacturing center *u* using transport mode *k* in time period *t*:

$$\sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR5_{muk}\, XTR5_{mukt}$$

[6] The transportation cost of the total number of FTL shipments from collection center *c* to remanufacturing center *u* using transport mode *k* in time period *t*:

$$\sum_{c=1}^{C}\sum_{u=1}^{U}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR6_{cuk}\, XTR6_{cukt}$$

[7] The transportation cost of the total number of FTL shipments from remanufacturing center *u* to dealership *d* using transport mode *k* in time period *t*:

$$\sum_{u=1}^{U}\sum_{d=1}^{D}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR7_{udk}\, XTR7_{udkt}$$

4.2.2.3 INVENTORY COSTS

Parameters:

$IVRS_s$:	Inventory carrying cost rate at the aluminum casting plant *s*
$IVRM_m$:	Inventory carrying cost rate at the engine plant *m*
$IVRA_a$:	Inventory carrying cost rate at the auto assembly plant *a*
$IVRU_u$:	Inventory carrying cost rate at the remanufacturing center *u*
PP_{in}:	Per unit price of the engine part *i* for engine type *n*
PNE_{nq}:	Per unit price of the new engine type *n* in product family *q*
PV_{nq}:	The average price of a new vehicle with engine type *n* in product family *q*
PNRE:	Per unit average price of rebuildable engines

PRE: Per unit average price of rebuilt engines

STRS: Number of days to keep inventories of aluminum ingots at the aluminum casting plant

STRRS: Number of days to keep inventories of recycled aluminum at the aluminum casting plant

STPS: Number of days to keep inventories of engine parts at the aluminum casting plant

STPM: Number of days to keep inventories of engine parts at the engine plants

STEM: Number of days to keep inventories of new engines at the engine plants

STEA: Number of days to keep inventories of new engines at the auto assembly plants

STVA: Number of days to keep inventories of new vehicles at the auto assembly plants

STEU: Number of days to keep inventories of rebuilt engines at the remanufacturing centers

Decision variables:

$XIV1_{st}$: Inventory of aluminum ingots, in lbs, held at the aluminum casting plant s at the end of time period t

$XIV2_{st}$: Inventory of recycled aluminum, in lbs, held at the aluminum casting plant s at the end of time period t

$XIV3_{inst}$: Number of units of engine part i for engine type n held as inventory at the aluminum casting plant s in time period t

$XIV4_{inmt}$: Number of units of aluminum engine part i for engine type n held as inventory at the engine plant m at the end of time period t

$XIV5_{nqmt}$: Number of units of engine type n in product family q held as inventory at the engine plant m at the end of time period t

$XIV6_{nqat}$: Number of units of engine type n in product family q held as inventory at the auto assembly plant a at the end of time period t

$XIV7_{nqat}$: Number of units of vehicles with engine type n in product family q held as inventory at the auto assembly plant a at the end of time period t

$XIV8_{ut}$: Number of units of rebuildable engines held at the remanufacturing center u at the end of time period t

$XIV9_{ut}$: Number of units of rebuilt engines held at the remanufacturing center u at the end of time period t

[1] Inventory cost of aluminum ingots at the aluminum casting plant s equals the amount of aluminum ingots multiplied by the per unit weight price of ingots, multiplied by the number of days ingots are kept as inventories, multiplied by the inventory carrying cost rate at the plant in time period t.

$$\sum_{s=1}^{S}\sum_{t=1}^{T} CP1 \times IVRS_s \times STRS \times XIV1_{st}$$

[2] Inventory cost of recycled aluminum at the aluminum casting plant s equals the amount of recycled aluminum multiplied by the per unit weight price of recycled aluminum, multiplied by the number of days recycled aluminum is kept as inventories, multiplied by the inventory carrying cost rate at the plant in time period t.

$$\sum_{s=1}^{S}\sum_{t=1}^{T} CP2 \times IVRS_s \times STRRS \times XIV2_{st}$$

[3] Inventory cost of engine part i at aluminum casting plant s equals the number of engine part i for engine type n held at the plant multiplied by the per unit price of engine part i for engine type n, multiplied by the number of days engine parts are kept at the plant, multiplied by the inventory carrying cost rate at the plant in time period t.

$$\sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{s=1}^{S}\sum_{t=1}^{T} PP_{in} IVRS_s STPS \times XIV3_{inst}$$

[4] Inventory cost of engine part i at engine plant m equals the number of engine part i for engine type n held at engine plant m multiplied by the per unit price of engine part i for engine type n, multiplied by the number of days engine parts are kept at the plant, multiplied by the inventory carrying cost rate at the plant in time period t.

$$\sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{t=1}^{T} PP_{in} IVRM_m STPM \times XIV4_{inmt}$$

[5] Inventory cost of engine type n in product family q at engine plant m equals the number of engine type n in product family q, multiplied by the per unit price of engine type n in

product family q, multiplied by the number of days new engines are kept at the engine plant, multiplied by the inventory carrying cost rate at the plant in time period t.

$$\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{m=1}^{M}\sum_{t=1}^{T} PNE_{nq} IVRM_m STEM \times XIV5_{nqmt}$$

[6] Inventory cost of engine type n in product family q at auto assembly plant a equals the number of engine type n in product family q, multiplied by the per unit price of engine type n in product family q, multiplied by the number of days new engines are kept at the plant, multiplied by the inventory carrying cost rate at the plant in time period t.

$$\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{a=1}^{A}\sum_{t=1}^{T} PNE_{nq} IVRA_a STEA \times XIV6_{nqat}$$

[7] Inventory cost of vehicles with engine type n in product family q at auto assembly plant a equals the number of vehicles with engine type n in product family q, multiplied by the per unit price of the vehicle with engine type n in product family q, multiplied by the number of days new vehicles are kept at the plant, multiplied by the inventory carrying cost rate at the plant in time period t.

$$\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{a=1}^{A}\sum_{t=1}^{T} PV_{nq} IVRA_a STVA \times XIV7_{nqat}$$

[8] Inventory cost of rebuildable engines at the remanufacturing center u equals the number of rebuildable engines, multiplied by the per unit price of rebuildable engines, multiplied by the number of days rebuildable engines are kept at the remanufacturing center, multiplied by the inventory carrying cost rate at the center in time period t.

$$\sum_{u=1}^{U}\sum_{t=1}^{T} PNRE \times IVRU_u STNEU \times XIV8_{ut}$$

[9] Inventory cost of rebuilt engines at the remanufacturing center u equals the number of rebuilt engines, multiplied by the per unit price of rebuilt engines, multiplied by the number of days rebuilt engines are kept at the remanufacturing center, multiplied by the inventory carrying cost rate at the center in time period t.

$$\sum_{u=1}^{U}\sum_{t=1}^{T} PRE \times IVRU_u STEU \times XIV9_{ut}$$

4.2.2.4 IN-TRANSIT INVENTORY COSTS

Parameters:

$LT1_{smk}$: Transportation lead time from aluminum casting plant *s* to engine plant *m* using transport mode *k* (days)

$LT2_{mak}$: Transportation lead time from engine plant *m* to auto assembly plant *a* using transport mode *k* (days)

$LT3_{adk}$: Transportation lead time from auto assembly plant *a* to dealership *d* using transport mode *k* (days)

$LT4_{crk}$: Transportation lead time from collection center *c* to recycling center *r* using transport mode *k* (days)

$IVRC_c$: Inventory carrying cost rate at collection center *c*

PFH: Per unit average price of a flattened hulk

Decision variables:

$XIT1_{sinmkt}$: Number of units of engine part *i* for engine type *n* in transit between aluminum casting plant *s* and engine plant *m* using transport mode *k* in time period *t*

$XIT2_{mnqakt}$: Number of units of engine type *n* in product family *q* in transit between engine plant *m* and auto assembly plant *a* using transport mode *k* in time period *t*

$XIT3_{anqdkt}$: Number of units of vehicles with engine type *n* in product family *q* in transit between auto assembly plant *a* and dealership *d* using transport mode *k* in the time period *t*

$XIT4_{crkt}$: Number of units flattened hulks in transit between collection center *c* and recycling center *r* using transport mode *k* in the time period *t*

[1] In-transit inventory cost of engine part *i* for engine type *n* from aluminum casting plant *s* to engine plant *m* equals the number of units of engine part *i* for engine type *n* in transit, multiplied by the per unit price of engine part *i* for engine type *n*, multiplied by the inventory carrying cost rate at the casting plant, multiplied by the transportation lead time from the casting plant to the engine plant using transport mode *k,* in time period *t*.

$$\sum_{s=1}^{S}\sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{t=1}^{T} PP_{in} IVRS_s LT1_{smk} XIT1_{\sin mkt}$$

[2] In-transit inventory cost of engine type *n* in product family *q* from engine plant *m* to auto assembly plant *a* equals the number of units of engine type *n* in product family *q* in transit, multiplied by the per unit price of engine type *n* in product family *q*, multiplied by the inventory carrying cost rate at the engine plant, multiplied by the transportation lead time from the engine plant to the auto assembly plant using transport mode *k,* in time period *t*.

$$\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{a=1}^{A}\sum_{k=1}^{K}\sum_{t=1}^{T} PNE_{nq} IVRM_m LT2_{mak} XIT2_{mnqakt}$$

[3] In-transit inventory cost of vehicles from auto assembly plant *a* to dealership *d* equals the number of units of vehicles with engine type *n* in product family *q* in transit, multiplied by the per unit price of the vehicle with engine type n in product family q, multiplied by the inventory carrying cost rate at the auto assembly plant, multiplied by the transportation lead time from the auto assembly plant to dealerships *d* using transport mode *k*, in time period *t*.

$$\sum_{a=1}^{A}\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{d=1d}^{D}\sum_{k=1}^{K}\sum_{t=1}^{T} PV_{nq} IVRA_a LT3_{adk} XIT3_{anqdkt}$$

[4] In-transit inventory cost of flattened hulks from collection center *c* to recycling center *r* equals the number of flattened hulks in transit, multiplied by the per unit price of a flattened hulk, multiplied by the inventory carrying cost rate at the collection center, multiplied by the transportation lead time from the collection center to the recycling center using transport mode *k*, in time period *t*.

$$\sum_{c=1}^{C}\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{t=1}^{T} PFH \times IVRC_c LT4_{crk} XIT4_{crkt}$$

4.2.2.5 LABOR COSTS

Parameters:

$CRLS_{st}$:	Regular labor cost per hour at aluminum casting plant *s* in time period *t*
$CRLM_{mt}$:	Regular labor cost per hour at engine plant *m* in time period *t*
$CRLU_{ut}$:	Regular labor cost per hour at remanufacturing center *u* in time period *t*
$COLS_{st}$:	Overtime labor cost per hour at aluminum casting plant *s* in time period *t*

$COLM_{mt}$:	Overtime labor cost per hour at engine plant *m* in time period *t*
$COLU_{ut}$:	Overtime labor cost per hour at remanufacturing center *u* in time period *t*
$CHLM_{mt}$:	Hiring cost per hour at engine plant *m* in time period *t*
$CLLM_{mt}$:	Layoff cost per hour at engine plant *m* in time period *t*

Decision variables:

$XRLS_{st}$:	Number of regular-time labor hours required at aluminum casting plant *s* in time period *t*
$XRLM_{mt}$:	Number of regular-time labor hours required at engine plant *m* in time period *t*
$XRLU_{ut}$:	Number of regular-time labor hours required at remanufacturing center *u* in time period *t*
$XOLS_{st}$:	Number of overtime labor hours required at aluminum casting plant *s* in time period *t*
$XOLM_{mt}$:	Number of overtime labor hours required at engine plant *m* in time period *t*
$XOLU_{ut}$:	Number of overtime labor hours required at remanufacturing center *u* in time period *t*
$XHLM_{mt}$:	Number of additional labor hours acquired at engine plant *m* in time period *t* through hiring
$XLLM_{mt}$:	Number of labor hours lost at engine plant *m* in time period *t* through layoffs

[1] At the aluminum casting plant *s*, the regular-time labor cost equals the number of regular time labor hours required at the plant in time period *t*, multiplied by the labor cost per hour at the plant.

$$\sum_{s=1}^{S}\sum_{t=1}^{T} CRLS_{st} XRLS_{st}$$

[2] At the aluminum casting plant *s*, the overtime labor cost equals the number of overtime labor hours required at the plant in time period *t*, multiplied by the overtime labor cost per hour at the plant.

$$\sum_{s=1}^{S}\sum_{t=1}^{T} COLS_{st} XOLS_{st}$$

[3] At the engine plant *m*, the regular-time labor cost equals the number of regular time labor hours required at the plant in time period *t*, multiplied by the labor cost per hour at the plant.

$$\sum_{m=1}^{M}\sum_{t=1}^{T} CRLM_{mt} XRLM_{mt}$$

[4] At the engine plant *m*, the overtime labor cost equals the number of overtime labor hours required at the plant in time period *t*, multiplied by the overtime labor cost per hour at the plant.

$$\sum_{m=1}^{M}\sum_{t=1}^{T} COLM_{mt} XOLM_{mt}$$

[5] At the engine plant *m*, the cost of hiring additional labor equals the number of additional labor hours required at the plant in time period *t*, multiplied by the per unit cost of acquiring additional labor hours at the plant.

$$\sum_{m=1}^{M}\sum_{t=1}^{T} CHLM_{mt} XHLM_{mt}$$

[6] At the engine plant *m*, the cost of laying off labor equals the number of labor hours reduced through layoff in time period *t*, multiplied by the corresponding per unit cost at the plant.

$$\sum_{m=1}^{M}\sum_{t=1}^{T} CLLM_{mt} XLLM_{mt}$$

[7] At the remanufacturing center *u*, the regular-time labor cost equals the number of regular time labor hours required at the center in time period *t*, multiplied by the regular time labor cost per hour at the center.

$$\sum_{u=1}^{U}\sum_{t=1}^{T} CRLU_{ut} XRLU_{ut}$$

[8] At the remanufacturing center u, the overtime labor cost equals the number of overtime labor hours required at the center in time period t, multiplied by the overtime labor cost per hour at the center.

$$\sum_{u=1}^{U}\sum_{t=1}^{T} COLU_{ut} XOLU_{ut}$$

4.2.2.6 HANDLING COSTS

Parameters:

$CH1_{nqa}$:	Cost of handling a unit of vehicle with engine type n in product family q at auto assembly plant a
$CH2_c$:	Cost of handling a unit of end-of-life vehicle at collection center c
ELV_{ct}:	Number of end-of-life vehicles collected at collection center c in time period t (equals to the expected number of vehicles retired over time multiplied by Ford's market share [1])
WRT:	The number of engine warranty replacements (units)

Decision variables:

$XPS2_{mnqakt}$:	Number of engine type n in product family q sent from engine plant m to auto assembly plant a using transport mode k in time period t

[1] At the auto assembly plant a, the total handling cost equals the number of units of engine type n in product family q shipped from engine plant m to auto assembly plant a using transport mode k in time period t, multiplied by the handling cost of a unit of vehicle with engine type n in product family q at the auto assembly plant.

$$\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{a=1}^{A}\sum_{k=1}^{K}\sum_{t=1}^{T} CH1_{nqa} XPS2_{mnqakt}$$

[2] At the collection center c, the total handling cost equals the number of end-of-life vehicles collected at the center in time period t, multiplied by the handling cost for a unit end-of-life vehicle at collection center c multiplied by the number of end-of-life vehicles and engine warranty returns collected at the center.

[1] Information is available at http://www.ford.com/en/company/about/sustainability/report/proData.htm#B

$$\sum_{c=1}^{C}\sum_{t=1}^{T}(ELV_{ct}+WRT)$$

4.2.2.7 REVENUES

Parameters:

$CP3_{miuk}$: The price of a unit of engine part i=3 shipped from engine plant m to remanufacturing center u using transport mode k

Decision variables:

$XPS5_{miukt}$: Number of engine part i=3 shipped from engine plant m to remanufacturing center u using transport mode k in time period t

[1] The revenue from selling engine parts to remanufacturing center u equals the number of engine part i=3 shipped from engine plant m to remanufacturing center u using transport mode k in period t, multiplied by the unit price of engine part i=3 at engine plant m.

$$\sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{u=1}^{U}\sum_{k=1}^{K}\sum_{t=1}^{T}CP3_{miuk}\,XPS5_{miukt}$$

4.2.3 The constraints

4.2.3.1 PRODUCTION AND PROCESS CAPACITY RESTRICTIONS

Parameters:

$PC1_{st}$: Production capacity at aluminum casting plant s in time period t

$PC2_{mt}$: Production capacity at engine plant m in time period t

$PC3_{at}$: Production capacity at auto assembly plant a in time period t

$PC4_{ct}$: Process capacity at collection center c in time period t

$PC5_{ut}$: Production capacity at remanufacturing center u in time period t

Decision variables:

$XPS1_{sinmkt}$: Number of engine parts i for engine type n produced at aluminum casting plant s and sent to engine plant m using transport mode k in time period t

$XPS3_{anqdkt}$: Number of vehicles with engine type *n* in product family *q* sent from auto assembly plant *a* to dealership *d* using transport mode *k* in time period *t*

$XPS4_{crkt}$: Number of flattened hulks sent from collection center *c* to recycling center *r* using transport mode *k* in time period *t*

$XPS7_{udkt}$: Number of rebuilt engines sent from remanufacturing center *u* to dealership *d* using transport mode *k* in time period *t*

[1] At the aluminum casting plant *s*, the number of engine parts *i* for engine type *n* shipped from the plant to engine plant *m* using transport mode *k* in time period *t* may not exceed the production capacity of the plant in time period *t*.

$$\sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{K} XPS1_{sinmkt} \leq PC1_{st} \qquad \forall s, t \quad \dots(1)$$

[2] At the engine plant *m*, the number of engines of type *n* in product family *q* shipped from the plant to the auto assembly plant *a* using transport mode *k* in time period *t* may not exceed the production capacity of the plant in time period *t*.

$$\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{a=1}^{A}\sum_{k=1}^{K} XPS2_{mnqakt} \leq PC2_{mt} \qquad \forall m, t \quad \dots(2)$$

[3] At the auto assembly plant *a*, the number of vehicles with engine type *n* in product family *q* shipped from the plant to dealership *d* using transport mode *k* in time period *t* may not exceed the production capacity of the plant in time period *t*.

$$\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{d=1}^{D}\sum_{k=1}^{K} XPS3_{anqdkt} \leq PC3_{at} \qquad \forall a, t \quad \dots(3)$$

[4] At the collection center *c*, the number of flattened hulks transported from the center to the recycling center *r* using transport mode *k* in time period *t* may not exceed the process capacity of the center in time period *t*.

$$\sum_{r=1}^{R}\sum_{k=1}^{K} XPS4_{crkt} \leq PC4_{ct} \qquad \forall c, t \quad \dots(4)$$

[5] At the remanufacturing center *u*, the number of rebuilt engines transported from the center to the dealership *d* using transport mode *k* in time period *t* may not exceed the production capacity of the center in time period *t*.

$$\sum_{d=1}^{D}\sum_{k=1}^{K} XPS7_{udkt} \leq PC5_{ut} \qquad \forall u, t \quad \ldots(5)$$

4.2.3.2 STORAGE SPACE CAPACITY RESTRICTIONS

Parameters:

$FC1_{st}$:	Storage space for holding aluminum ingots at aluminum casting plant s in time period t (cu.ft.)
$FC2_{st}$:	Storage space for holding recycled aluminum at aluminum casting plant s at in time period t (cu.ft.)
$FC3_{inst}$:	Storage space for holding engine part i for engine type n at aluminum casting plant s in time period t (cu.ft./unit)
$FC4_{inmt}$:	Storage space for holding engine part i for engine type n at engine plant m in time period t (cu.ft./unit)
$FC5_{nqmt}$:	Storage space for holding engine type n in product family q at engine plant m in time period t (cu.ft./unit)
$FC6_{nqat}$:	Storage space for holding engine type n in product family q at auto assembly plant a in time period t (cu.ft./unit)
$FC7_{nqat}$:	Storage space for holding vehicles with engine type n in product family q at auto assembly plant a in time period t (sq.ft./unit)
$FC8_{ut}$:	Storage space for holding rebuildable engines at remanufacturing center u in time period t (cu.ft.)
$FC9_{ut}$:	Storage space for holding rebuilt engines at remanufacturing center u in time period t (cu.ft.)
$VP1$:	The volume of a unit weight of aluminum ingot (cu.ft./lb)
$VP2$:	The volume of a unit weight of recycled aluminum (cu.ft./lb)
$VP3_{nq}$:	The volume of a unit of engine type n in product family q (cu.ft./unit)
$VP4_{in}$:	The volume of a unit of engine part i for engine type n (cu.ft./unit)
$VP5$:	The average amount of floor space a vehicle occupies (sq.ft./unit)
$VP7$:	The average volume of an engine (cu.ft./unit)

[1] At the aluminum casting plant *s*, the amount of space needed by the inventory of the aluminum ingots in time period *t* may not exceed the storage capacity at the plant.

$$VP1 \times XIV1_{st} \leq FC1_{st} \qquad \forall s, t \quad \ldots(6)$$

[2] At the aluminum casting plant *s*, the amount of space needed by the inventory of the recycled aluminum in time period *t* may not exceed the storage capacity at the plant.

$$VP2 \times XIV2_{st} \leq FC2_{st} \qquad \forall s, t \quad \ldots(7)$$

[3] At the aluminum casting plant *s*, the amount of space needed by the inventory of engine part *i* engine type *n* in time period *t* may not exceed the allotted storage at the plant.

$$\sum_{i=1}^{I}\sum_{n=1}^{N} VP4_{in} XIV3_{inst} \leq \sum_{i=1}^{I}\sum_{n=1}^{N} FC3_{inst} \qquad \forall s, t \quad \ldots(8)$$

[4] At the engine plant *m*, the amount of space needed by the inventory of engine part *i* engine type *n* in time period *t* may not exceed allotted storage space at the plant.

$$\sum_{i=1}^{I}\sum_{n=1}^{N} VP4_{in} XIV4_{inmt} \leq \sum_{i=1}^{I}\sum_{n=1}^{N} FC4_{inmt} \qquad \forall m, t \quad \ldots(9)$$

[5] At the engine plant *m*, the amount of space needed by the inventory of engine type *n* in product family *q* in time period *t* may not exceed the allotted storage space at the plant.

$$\sum_{n=1}^{N}\sum_{q=1}^{Q} VP3_{nq} XIV5_{nqmt} \leq \sum_{n=1}^{N}\sum_{q=1}^{Q} FC5_{nqmt} \qquad \forall m, t \quad \ldots(10)$$

[6] At the auto assembly plant *a*, the amount of space needed by the inventory of engine type *n* in product family *q* in time period *t* should be within the allotted storage space at the plant.

$$\sum_{n=1}^{N}\sum_{q=1}^{Q} VP3_{nq} XIV6_{nqat} \leq \sum_{n=1}^{N}\sum_{q=1}^{Q} FC6_{nqat} \qquad \forall a, t \quad \ldots(11)$$

[7] At the auto assembly plant *a*, the amount of space needed by the inventory of vehicles with engine type *n* in product family *q* in time period *t* should be within the allotted storage space at the plant.

$$\sum_{n=1}^{N}\sum_{q=1}^{Q} VP5XIV7_{nqat} \leq \sum_{n=1}^{N}\sum_{q=1}^{Q} FC7_{nqat} \qquad \forall a, t \quad \ldots(12)$$

[8] At the remanufacturing center *u*, the amount of space needed by the inventory of rebuildable engines in time period *t* may not exceed the storage capacity at the center.

$$VP7 \times XIV8_{ut} \leq FC8_{ut} \qquad \forall u, t \quad \ldots(13)$$

[9] At the remanufacturing center u, the amount of space needed by the inventory of rebuilt engines in time period t may not exceed the storage capacity at the center.

$$VP7 \times XIV9_{ut} \leq FC9_{ut} \qquad \forall u, t \quad \ldots(14)$$

4.2.3.3 PRODUCTION LABOR HOUR RESTRICTIONS

Parameters:

RHS_{ins}: Per unit production time of engine part i for engine type n at the aluminum casting plant s (in hrs.)

RHM_{nqm}: Per unit production time of engine type n in product family q at the engine plant m (in hrs.)

RHU_u: Per unit time to rebuild engines at the remanufacturing center u (in hrs.)

$MLHS_{st}$: Maximum limit of labor hours allowed at the aluminum casting plant s in time period t

$MLHM_{mt}$: Maximum limit of labor hours allowed at the engine plant m in time period t

$MLHU_{ut}$: Maximum limit of labor hours allowed at the remanufacturing center u in time period t

fs_s: Ratio of overtime labor hours to regular time labor hours at the aluminum casting plant s

fm_m: Ratio of overtime labor hours to regular time labor hours at the engine plant m

fu_u: Ratio of overtime labor hours to regular time labor hours at the remanufacturing center u

Decision variables:

XM_{nqmt}: Number of units of engine type n in product family q produced at engine plant m in time period t

$XPS6_{cukt}$: Number of rebuildable engines processed at collection center c and sent to remanufacturing center u using transport mode k in time period t

[1] The regular time labor hours at the aluminum casting plant *s* in time period *t* should not exceed the maximum allowable limit.

$$XRLS_{st} \leq MLHS_{st} \qquad \forall s, t \quad ...(15)$$

[2] The overtime labor hours at the aluminum casting plant *s* in time period *t* are normally represented as a percentage of the regular time hours.

$$XOLS_{st} \leq fs_s XRLS_{st} \qquad \forall s, t \quad ...(16)$$

[3] At the aluminum casting plant *s*, the regular and overtime labor hours together should be sufficient to meet the production requirement.

$$XRLS_{st} + XOLS_{st} \geq \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{K} RHS_{ins} XPS1_{sinmkt} \qquad \forall s, t \quad ...(17)$$

[4] The regular time labor hours at the engine plant *m* in time period *t* should not exceed the maximum allowable limit.

$$XRLM_{mt} \leq MLHM_{mt} \qquad \forall m, t \quad ...(18)$$

[5] The overtime labor hours at the engine plant *m* in time period *t* are normally expressed as a percentage of the regular time hours.

$$XOLM_{mt} \leq fm_m XRLM_{mt} \qquad \forall m, t \quad ...(19)$$

[6] At the engine plant *m*, the regular and overtime labor hours, plus any additional labor hours acquired through hiring, minus any labor hour reductions through layoffs should be sufficient to meet the production requirement.

$$XRLM_{mt} + XOLM_{mt} + XHLM_{mt} - XLLM_{mt} \geq \sum_{n=1}^{N}\sum_{q=1}^{Q} RHM_{nqm} XM_{nqmt} \qquad \forall m, t \quad ...(20)$$

[7] At the remanufacturing center *u* in time period *t*, the regular time labor hours utilized should not exceed the maximum allowable limit.

$$XRLU_{ut} \leq MLHU_{ut} \qquad \forall u, t \quad ...(21)$$

[8] At the remanufacturing center *u* in time period *t*, the overtime labor hours utilized are normally represented as a percentage of the regular time labor hours.

$$XOLU_{ut} \leq fu_u XRLU_{ut} \qquad \forall u, t \quad ...(22)$$

[9] At the remanufacturing center *u* in time period *t*, the regular time and the overtime labor hours together should be sufficient to meet the production requirement.

$$XRLU_{ut} + XOLU_{ut} \geq \sum_{c=1}^{C}\sum_{k=1}^{K} RHU_u XPS6_{cukt} \qquad \forall u, t \quad ...(23)$$

4.2.3.4 TRANSPORT CARRIERS' CAPACITY RESTRICTIONS

Parameters:

NN_{nqk}:	Number of engines of type n in product family q that can be loaded in one FTL of transport mode k
NNN_k:	Number of vehicles that can be loaded in one FTL of transport mode k
$WP1$:	The average weight of engine part i=3
$WP2$:	The average weight of an engine
$WP3_{in}$:	The weight of one unit of engine part i=1 for engine type n
$WP4$:	The average weight of a flattened hulk
WC_k:	The weight capacity of transport mode k
VC_k:	The volume capacity of transport mode k
$VP6$:	The average volume of a flattened hulk (cu.ft./unit)
$VP8$:	The average volume of an engine part i=3 sold from engine plants (cu.ft/unit)

[1a] At the aluminum casting plant s, in time period t, the total weight of all engine parts i for engine type n shipped to the engine plant m should not exceed the available transport carriers' weight capacity (which equals the number of FTL shipments using transport mode k multiplied by the weight capacity of transport mode k).

$$\sum_{i=1}^{I}\sum_{n=1}^{N} WP3_{in}\, XPS1_{sin\,mkt} \leq WC_k\, XTR1_{smkt} \qquad \forall s, m, k, t$$

or,

$$XTR1_{smkt} \geq \frac{\sum_{i=1}^{I}\sum_{n=1}^{N} WP3_{in}\, XPS1_{sin\,mkt}}{WC_k} \qquad \forall s, m, k, t$$

[1b] At the aluminum casting plant s, in time period t, the total volume of all engine parts i for engine type n shipped to the engine plant m should not exceed the available transport carriers' volume capacity (which equals the number of FTL shipments using transport mode k multiplied by the volume capacity of transport mode k).

$$\sum_{i=1}^{I}\sum_{n=1}^{N} VP4_{in}\, XPS1_{sin\,mkt} \leq VC_k\, XTR1_{smkt} \qquad \forall s, m, k, t$$

or,

$$XTR1_{smkt} \geq \frac{\sum_{i=1}^{I}\sum_{n=1}^{N} VP4_{in} XPS1_{\sin mkt}}{VC_k} \qquad \forall s, m, k, t$$

[1] The number of required FTL shipments between the aluminum casting plant s and the engine plant m is the larger of two numbers computed in [1a] and [1b] above.

$$XTR1_{smkt} \geq \max\left(\frac{\sum_{i=1}^{I}\sum_{n=1}^{N} WP3_{in} XPS1_{\sin mkt}}{WC_k}, \frac{\sum_{i=1}^{I}\sum_{n=1}^{N} VP4_{in} XPS1_{\sin mkt}}{VC_k}\right) \qquad \forall s, m, k, t \ldots(24)$$

[2] At the engine plant m, in time period t, the number of FTL shipments to the auto assembly plant a using transport mode k should be greater than or equal the number of units of engine type n in product family q to be transported from engine plant m to auto assembly plant a, divided by the number of engines of type n in product family q that can be loaded in one FTL of transport mode k.

$$XTR2_{makt} \geq \frac{\sum_{n=1}^{N}\sum_{q=1}^{Q} XPS2_{mnqakt}}{\sum_{n=1}^{N}\sum_{q=1}^{Q} NN_{nqk}} \qquad \forall m, a, k, t \ \ldots(25)$$

[3] Similarly, at the auto assembly plant a, the number of FTL shipments to the dealership d using transport mode k should be greater than or equal the number of vehicles with engine type n in product family q to be transported from auto assembly plant a to dealership d, divided by the number of vehicles that can be loaded in one FTL of transport mode k.

$$XTR3_{adkt} \geq \frac{\sum_{n=1}^{N}\sum_{q=1}^{Q} XPS3_{anqdkt}}{NNN_k} \qquad \forall a, d, k, t \ \ldots(26)$$

[4a] At the collection center c, in time period t, the total weight of all the flattened hulks shipped to the recycling center r should not exceed the available transport carriers' weight capacity (which equals the number of FTL shipments using transport mode k multiplied by the weight capacity of transport mode k).

$$WP4 \times XPS4_{crkt} \leq WC_k XTR4_{crkt} \qquad \forall c, r, k, t$$

or,

$$XTR4_{crkt} \geq \frac{WP4 \times XPS4_{crkt}}{WC_k} \qquad \forall c, r, k, t$$

[4b] Similarly, at the collection center *c*, in time period *t*, the total volume of all the flattened hulks shipped to the recycling center *r* should not exceed the available transport carriers' volume capacity (which equals the number of FTL shipments using transport mode *k* multiplied by the volume capacity of transport mode *k*).

$$VP6 \times XPS4_{crkt} \leq VC_k XTR4_{crkt} \qquad \forall c, r, k, t$$

or,

$$XTR4_{crkt} \geq \frac{VP6 \times XPS4_{crkt}}{VC_k} \qquad \forall c, r, k, t$$

[4] The number of required FTL shipments between the collection center *c* and the recycling center *r* is the larger of two numbers computed in [4a] and [4b] above.

$$XTR4_{crkt} \geq \max\left(\frac{WP4 \times XPS4_{crkt}}{WC_k}, \frac{VP6 \times XPS4_{crkt}}{VC_k}\right) \qquad \forall c, r, k, t \quad \ldots(27)$$

[5a] At the engine plant *m*, the total weight of engine parts *i*=3 to be transported to remanufacturing center *u* should be within the transport carriers' weight capacity (which equals the number of FTL shipments using transport mode *k* multiplied by the weight capacity of transport mode *k*).

$$\sum_{i=1}^{I}\sum_{t=1}^{T} WP1 \times XPS5_{miukt} \leq \sum_{t=1}^{T} WC_k XTR5_{mukt} \qquad \forall m, i, u, k$$

or,

$$\sum_{t=1}^{T} XTR5_{mukt} \geq \frac{\sum_{i=1}^{I}\sum_{t=1}^{T} WP1 \times XPS5_{miukt}}{WC_k} \qquad \forall m, i, u, k$$

[5b] Similarly, at the engine plant *m*, the total volume of engine parts *i*=3 to be transported to remanufacturing center *u* should be within the transport carriers' volume capacity (which equals the number of FTL shipments using transport mode *k* multiplied by the volume capacity of transport mode *k*).

$$\sum_{i=1}^{I}\sum_{t=1}^{T} VP8 \times XPS5_{miukt} \leq \sum_{t=1}^{T} VC_k XTR5_{mukt} \qquad \forall m, i, u, k$$

or,

$$\sum_{t=1}^{T} XTR5_{mukt} \geq \frac{\sum_{i=1}^{I}\sum_{t=1}^{T} VP8 \times XPS5_{miukt}}{VC_k} \qquad \forall m, i, u, k$$

[5] The number of required FTL shipments between the engine plant *m* and the remanufacturing center *u* is the larger of two numbers computed in [5a] and [5b] above.

$$\sum_{t=1}^{T} XTR5_{mukt} \geq \max\left(\frac{\sum_{i=1}^{I}\sum_{t=1}^{T} WP1 \times XPS5_{miukt}}{WC_k}, \frac{\sum_{i=1}^{I}\sum_{t=1}^{T} VP8 \times XPS5_{miukt}}{VC_k} \right) \qquad \forall m, i, u, k \ldots (28)$$

[6a] At the collection center *c*, the total weight of end-of-life engines to be transported to the remanufacturing center *u* should be within the transport carriers' weight capacity (which equals the number of FTL shipments using transport mode *k* multiplied by the weight capacity of transport mode *k*).

$$\sum_{t=1}^{T} WP2 \times XPS6_{cukt} \leq \sum_{t=1}^{T} WC_k XTR6_{cukt} \qquad \forall c, u, k$$

or,

$$\sum_{t=1}^{T} XTR6_{cukt} \geq \frac{\sum_{t=1}^{T} WP2 \times XPS6_{cukt}}{WC_k} \qquad \forall c, u, k$$

[6b] At the collection center *c*, the total volume of end-of-life engines to be transported to the remanufacturing center *u* should be within the transport carriers' volume capacity (which equals the number of FTL shipments using transport mode *k* multiplied by the volume capacity of transport mode *k*).

$$\sum_{t=1}^{T} VP7 \times XPS6_{cukt} \leq \sum_{t=1}^{T} VC_k XTR6_{cukt} \qquad \forall c, u, k$$

or,

$$\sum_{t=1}^{T} XTR6_{cukt} \geq \frac{\sum_{t=1}^{T} VP7 \times XPS6_{cukt}}{VC_k} \qquad \forall c, u, k$$

[6] The number of required FTL shipments between the collection center *c* and the remanufacturing center *u* is the larger of the two numbers computed in [6a] and [6b] above.

$$\sum_{t=1}^{T} XTR6_{cukt} \geq \max\left(\frac{\sum_{t=1}^{T} WP2 \times XPS6_{cukt}}{WC_k}, \frac{\sum_{t=1}^{T} VP7 \times XPS6_{cukt}}{VC_k}\right) \quad \forall c, u, k \ldots(29)$$

[7a] At the remanufacturing center u, the total weight of the rebuilt engines to be transported to dealership d should be within the transport carriers' weight capacity (which equals the number of FTL shipments using transport mode k multiplied by the weight capacity of transport mode k).

$$\sum_{t=1}^{T} WP2 \times XPS7_{udkt} \leq \sum_{t=1}^{T} WC_k XTR7_{udkt} \quad \forall u, d, k$$

$$\sum_{t=1}^{T} XTR7_{udkt} \geq \frac{\sum_{t=1}^{T} WP2 \times XPS7_{udkt}}{WC_k} \quad \forall u, d, k$$

[7b] At the remanufacturing center u, the total volume of the rebuilt engines to be transported to dealership d should be within the transport carriers' volume capacity (which equals the number of FTL shipments using transport mode k multiplied by the volume capacity of transport mode k).

$$\sum_{t=1}^{T} VP7 \times XPS7_{udkt} \leq \sum_{t=1}^{T} VC_k XTR7_{udkt} \quad \forall u, d, k$$

$$\sum_{t=1}^{T} XTR7_{udkt} \geq \frac{\sum_{t=1}^{T} VP7 \times XPS7_{udkt}}{VC_k} \quad \forall u, d, k$$

[7] The number of required FTL shipments between the remanufacturing center u and the dealership d is the larger of two numbers computed in [7a] and [7b] above.

$$\sum_{t=1}^{T} XTR7_{udkt} \geq \max\left(\frac{\sum_{t=1}^{T} WP2 \times XPS7_{udkt}}{WC_k}, \frac{\sum_{t=1}^{T} VP7 \times XPS7_{udkt}}{VC_k}\right) \quad \forall u, d, k \ldots(30)$$

4.2.3.5 INVENTORY CAPACITY RESTRICTIONS

I. INVENTORY BALANCE:

Parameters:

BOM_{in}: Bill of material utilization rate of engine part *i* per unit of engine type *n*

$AD9_{nqdt}$: Number of vehicles with engine type *n* in product family *q* needed at the dealership *d* to sell to customers in time period *t*

$AD10_{dt}$: Expected number of rebuilt engines needed at dealership *d* in time period *t*

$LT5_{muk}$: Transportation lead time from engine plant *m* to remanufacturing center *u* using transport mode *k* (days)

$LT6_{cuk}$: Transportation lead time from collection center *c* to remanufacturing center *u* using transport mode *k* (days)

$LT7_{udk}$: Transportation lead time from remanufacturing center *u* to dealership *d* using transport mode *k* (days)

Decision variables:

$XUS1_{st}$: Amount of aluminum used at aluminum casting plant *s* in time period *t*

$XUS2_{inmt}$: Number of units of engine part *i* for engine type *n* used at the engine plant *m* in time period *t*

$XUS3_{nqat}$: Number of units of engine type *n* in product family *q* used in the production of vehicles at auto assembly plant *a* in time period *t*

$XUS4_{ut}$: Number of rebuildable engines processed at remanufacturing center *u* in time period *t*

$XM1_{inst}$: Number of units of engine part *i* for engine type *n* produced at aluminum casting plant *s* in time period *t*

[1] At the aluminum casting plant *s*, in time period *t*, the aluminum inventory is equal to the previous period's inventory plus the amount of aluminum ingots purchased from the suppliers, plus the amount of recycled aluminum purchased from the recycling center, minus the amount of aluminum ingots and recycled aluminum used at the plant.

$$XIV1_{st} + XIV2_{st} = XIV1_{s(t-1)} + XIV2_{s(t-1)} + XC_{st} - XUS1_{st} \qquad \forall s, t \quad \ldots(31)$$

[2] At the aluminum casting plant, in time period *t*, the inventory of engine part *i* for engine type *n* is equal to the previous period's inventory plus the number of parts produced at the plant, minus the number of parts shipped to engine plant *m*.

$$XIV3_{inst} = XIV3_{ins(t-1)} + XM1_{inst} - \sum_{m=1}^{M}\sum_{k=1}^{K} XPS1_{sin\,mkt} \qquad \forall i,\ n,\ s,\ t \ \ldots(32)$$

[3] The material consumption at the aluminum casting plant *s*.

$$\sum_{i=1}^{I}\sum_{n=1}^{N} WP3_{in} XM1_{inst} = 0.90 XUS1_{st} \qquad \forall s,\ t \ \ldots(33)$$

[4] At the aluminum casting plant, in time period *t*, the number of units of engine part *i* for engine type *n* shipped to the engine plant *m* should be greater than or equal to the demand for the engine part *i* for engine type *n* at the engine plant.

$$\sum_{s=1}^{S}\sum_{k=1}^{K} XPS1_{sin\,mkt} \geq AD4_{inmt} \qquad \forall i,\ n,\ m,\ t \ \ldots(34)$$

[5] At the engine plant *m*, in time period *t*, the inventory of engine part *i* for engine type *n* is equal to the previous period's inventory, plus the number of parts received from the aluminum casting plant *s*, minus the number of the parts sent to the remanufacturing center *u*, minus the number of the parts used at the engine plant. When s=1, $XPS5_{miukt}$ is equal to zero.

$$XIV4_{inmt} = XIV4_{inm(t-1)} + \sum_{s=1}^{S}\sum_{k=1}^{K} XPS1_{sin\,mk(t-LT1_{smk})} - \sum_{u=1}^{U}\sum_{k=1}^{K} XPS5_{miukt} - XUS2_{inmt}$$

$\forall i,\ n,\ m,\ t \ \ldots(35)$

[6] At the engine plant *m*, in time period *t*, the inventory of engine type *n* in product family *q* is equal to the previous period's inventory, plus the number of units of the engine produced at the plant in that period, minus the number of units of the engine shipped to the auto assembly plant *a* using the transport mode *k*, minus the number of engine warranty replacements.

$$XIV5_{nqmt} = XIV5_{nqm(t-1)} + XM_{nqmt} - \sum_{a=1}^{A}\sum_{k=1}^{K} XPS2_{mnqakt} - WRT \qquad \forall n,\ q,\ m,\ t \ \ldots(36)$$

[7] The number of the units of engine part *i* for engine type *n* in product family *q* consumed at the engine plant *m*.

$$XUS2_{inmt} = BOM_{in} XM_{nqmt} \qquad \forall i,\ n,\ q,\ m,\ t \ \ldots(37)$$

[8] At the engine plant *m*, the number of the units of engine type *n* in product family *q* sent to auto assembly plant *a* in time period *t* should be greater than or equal to the demand for that engine type.

$$\sum_{m=1}^{M}\sum_{k=1}^{K} XPS2_{mnqakt} \geq AD6_{nqat} \qquad \forall n, q, m, t \ \ldots(38)$$

[9] At the auto assembly plant *a*, in time period *t*, the inventory of engine type *n* in product family *q* is equal to the previous period's inventory, plus number of the engines shipped from engine plant *m* to auto assembly plant *a* in that period, minus the number of units of the engine installed in vehicles.

$$XIV6_{nqat} = XIV6_{nqa(t-1)} + \sum_{m=1}^{M}\sum_{k=1}^{K} XPS2_{mnqak(t-LT2_{mak})} - XUS3_{nqat} \qquad \forall n, q, a, t \ \ldots(39)$$

[10] At the auto assembly plant *a*, in time period *t*, the inventory of vehicles with engine type *n* in product family *q* is equal to the previous period's inventory, plus the number of vehicles produced in that period, minus the number of vehicles with engine type *n* in product family *q* shipped from the plant to dealership *d*.

$$XIV7_{nqat} = XIV7_{nqa(t-1)} + XUS3_{nqat} - \sum_{d=1}^{D}\sum_{k=1}^{K} XPS3_{anqdkt} \qquad \forall n, q, a, t \ \ldots(40)$$

[11] The number of vehicles with engine type *n* in product family *q* shipped from the auto assembly plant *a* to dealership *d* using transport mode *k* in time period *t* should be greater than or equal to the specified demand.

$$\sum_{a=1}^{A}\sum_{k=1}^{K} XPS3_{anqdkt} \geq AD9_{nqdt} \qquad \forall n, q, d, t \ \ldots(41)$$

[12] At the collection center *c*, in time period *t*, the number of flattened hulks shipped out to the recycling centers is equal to the number of end-of-life vehicles collected at the center plus the number of engine warranty replacements.

$$\sum_{r=1}^{R}\sum_{k=1}^{K} XPS4_{crkt} = ELV_{ct} + WRT \qquad \forall c, t \ \ldots(42)$$

[13] The number of rebuildable engines shipped out from the collection center *c* to the remanufacturing center *u* in time period *t* is equal to 18%[2] of the number of engines collected at the center plus the number of engine warranty replacements.

$$\sum_{u=1}^{U}\sum_{k=1}^{K} XPS6_{cukt} = 18\%(ELV_{ct} + WRT) \qquad \forall c, t \ \ldots(43)$$

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[2] About 2.2 million engines are remanufactured annually, approximately 12 million vehicles retired per year

[14] At the remanufacturing center *u*, in time period *t*, the inventory of the number of rebuildable engines is equal to the previous period's inventory, plus the number of engines received from the collection center *c* during this time period, minus the number of rebuildable engines processed during time period *t*.

$$XIV8_{ut} = XIV8_{u(t-1)} + \sum_{c=1}^{C}\sum_{k=1}^{K} XPS6_{cuk(t-LT6_{cuk})} - XUS4_{ut} \qquad \forall u, t \quad ...(44)$$

[15] At the remanufacturing center *u*, in time period *t*, the inventory of the number of rebuilt engines is equal to the previous period's inventory, plus the number of engines rebuilt during time period *t*, minus the number of rebuilt engines sent to dealership *d*.

$$XIV9_{ut} = XIV9_{u(t-1)} + XUS4_{ut} - \sum_{d=1}^{D}\sum_{k=1}^{K} XPS7_{udkt} \qquad \forall u, t \quad ...(45)$$

[16] At the remanufacturing center *u*, in time period *t*, the number of units of engine part $i=3$ received from the engine plant *m* using transport mode *k* should be equal to the number of engines sent to dealership *d*.

$$\sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{k=1}^{K} XPS5_{miukt} = \sum_{d=1}^{D}\sum_{k=1}^{K} XPS7_{udkt} \qquad \forall u, t \quad ...(46)$$

[17] At the remanufacturing center *u*, in time period *t*, the number of rebuilt engines shipped out to dealership *d* using transport mode *k*, should be greater than or equal to the demand for rebuilt engines at dealership *d*.

$$\sum_{u=1}^{U}\sum_{k=1}^{K} XPS7_{udkt} \geq AD10_{dt} \qquad \forall d, t \quad ...(47)$$

II. IN-TRANSIT INVENTORY BALANCE

[1] The in-transit inventory of engine part *i* for engine type *n* between the aluminum casting plant *s* and the engine plant *m* using transport mode *k*, in time period *t*, is equal to the previous period's in-transit inventory, plus the number of units of engine part *i* for engine type *n* shipped out, minus any shipments received at the engine plant which were sent in time period $t\text{-}LT1_{smk}$, where $LT1_{smk}$ is the transportation lead time from aluminum casting plant *s* to engine plant *m*.

$$XIT1_{sinmkt} = XIT1_{sinmk(t-1)} + XPS1_{sinmkt} - XPS1_{sinmk(t-LT1_{smk})} \qquad \forall s, i, n, m, k, t \quad ...(48)$$

[2] The in-transit inventory of engine type *n* in product family *q* between the engine plant *m* and the auto assembly plant *a* using transport mode *k,* in time period *t*, is equal to the previous period's in-transit inventory, plus the number of units of engine type *n* in product family *q* shipped out, minus any shipments received at the assembly plant which were sent in time period $t\text{-}LT2_{mak}$, where $LT2_{mak}$ is the transportation lead time from engine plant *m* to auto assembly plant *a*.

$$XIT2_{mnqakt} = XIT2_{mnqak(t-1)} + XPS2_{mnqakt} - XPS2_{mnqak(t-LT2_{mak})} \qquad \forall m, n, q, a, k, t \quad ...(49)$$

[3] The in-transit inventory of vehicles with engine type *n* in product family *q* between the auto assembly plant *a* and dealership *d* using transport mode *k,* in time period *t*, is equal to the previous period's in-transit inventory, plus the number of vehicles with engine type *n* in product family *q* shipped to dealership *d*, minus any shipments received at the dealership which were sent in time period $t\text{-}LT3_{adk}$, where$LT3_{adk}$ is the transportation lead time from auto assembly plant *a* to dealership *d*.

$$XIT3_{anqdkt} = XIT3_{anqdk(t-1)} + XPS3_{anqdkt} - XPS3_{anqdk(t-LT3_{adk})} \qquad \forall a, n, q, d, k, t \quad ...(50)$$

[4] The in-transit inventory of flattened hulks between the collection center *c* and the recycling center *r* using transport mode *k,* in time period *t*, is equal to the previous period's in-transit inventory, plus the number of flattened hulks shipped to the recycling center *r*, minus any shipments received at the recycling center *r* which were sent in time period $t\text{-}LT4_{crk}$, where $LT4_{crk}$ is the transportation lead time from the collection center *c* to the recycling center *r*.

$$XIT4_{crkt} = XIT4_{crk(t-1)} + XPS4_{crkt} - XPS4_{crk(t-LT4_{crk})} \qquad \forall c, r, k, t \quad ...(51)$$

III. SAFETY STOCK:

Parameters:

$AD1_{st}$: Average demand for aluminum ingots at aluminum casting plant *s* in time period *t* (lbs/day)

$AD2_{st}$: Average demand for recycled aluminum at aluminum casting plant *s* in time period *t* (lbs/day)

$AD3_{inst}$: Average demand for engine part *i* for engine type *n* at aluminum casting plant *s* in time period *t* (units/day)

$AD4_{inmt}$: Average demand for engine part *i* for engine type *n* at engine plant *m* in time period *t* (units/day)

$AD5_{nqmt}$: Average demand for engine type *n* in product family *q* at engine plant *m* in time period *t* (units/day)

$AD6_{nqat}$: Average demand for engine type *n* in product family *q* at auto assembly plant *a* in time period *t* (units/day)

$AD7_{nqat}$: Average demand for vehicles with engine type *n* in product family *q* at auto assembly plant *a* in time period *t* (units/day)

$AD8_{ut}$: Average demand for rebuilt engines at remanufacturing center *u* in time period *t* (units/day)

Decision variables:

$SS1_{st}$: Safety stock of aluminum ingots at aluminum casting plant *s* in time period *t*

$SS2_{st}$: Safety stock of recycled aluminum at aluminum casting plant *s* in time period *t*

$SS3_{inst}$: Safety stock of engine part *i* for engine type *n* at aluminum casting plant *s* in time period *t*

$SS4_{inmt}$: Safety stock of engine part *i* for engine type *n* at engine plant *m* in time period *t*

$SS5_{nqmt}$: Safety stock of engine type *n* in product family *q* at engine plant *m* in time period *t*

$SS6_{nqat}$: Safety stock of engine type *n* in product family *q* at auto assembly plant *a* in time period *t*

$SS7_{nqat}$: Safety stock of vehicles with engine type *n* in product family *q* at auto assembly plant *a* in time period *t*

$SS8_{ut}$: Safety stock of rebuilt engines at remanufacturing center *u* at time period *t*

At present, the safety stock policy of Ford Motor Co. is to keep two days' worth of production at each stage of the production/assembly system. This policy is translated into a set of general constraints on safety stock levels, as described below.

[1] The safety stock of aluminum ingots at the aluminum casting plant *s* in time period *t* is equal to the average daily demand for aluminum ingots at the aluminum casting plant *s* multiplied by the number of days' worth of production specified by the policy at the aluminum casting plant *s*.

$$SS1_{st} = AD1_{st}STRS \qquad \forall s, t \quad \ldots(52)$$

Now, the inventory of aluminum ingots at the aluminum casting plant *s* in time period *t* should be greater than or equal to the safety stock $SS1_{st}$.

$$XIV1_{st} \geq SS1_{st} \qquad \forall s, t \quad \ldots(53)$$

[2] The safety stock of recycled aluminum at the aluminum casting plant *s* in time period *t* is equal to the average daily demand for recycled aluminum at the aluminum casting plant *s* multiplied by the number of days' worth of production specified by the policy at the aluminum casting plant *s*.

$$SS2_{st} = AD2_{st}STRRS \qquad \forall s, t \quad \ldots(54)$$

Now, the inventory of recycled aluminum at the aluminum casting plant *s* in time period *t* should be greater than or equal to the safety stock $SS2_{st}$.

$$XIV2_{st} \geq SS2_{st} \qquad \forall s, t \quad \ldots(55)$$

[3] The safety stock of engine part *i* for engine type *n* at the aluminum casting plant *s* in time period *t* is equal to the average daily demand for engine part *i* for engine type *n* at the aluminum casting plant *s* multiplied by the number of days' worth of production specified by the policy at the aluminum casting plant *s*.

$$SS3_{inst} = AD3_{inst}STPS \qquad \forall i, n, s, t \quad \ldots(56)$$

The inventory of engine part *i* for engine type *n* at the aluminum casting plant *s* in time period *t* should now be greater than or equal to the safety stock $SS3_{inst}$.

$$XIV3_{inst} \geq SS3_{inst} \qquad \forall i, n, s, t \quad \ldots(57)$$

[4] The safety stock of engine part *i* for engine type *n* at the engine plant *m* in time period *t* is equal to the average daily demand for engine part *i* for engine type *n* at the engine plant *m* multiplied by the number of days' worth of production specified by the policy at the engine plant *m*.

$$SS4_{inmt} = AD4_{inmt}STPM \qquad \forall i, n, m, t \quad ...(58)$$

The inventory of engine part *i* for engine type *n* at engine plant *m* in time period *t* should now be greater than or equal to the safety stock $SS4_{inmt}$.

$$XIV4_{inmt} \geq SS4_{inmt} \qquad \forall i, n, m, t \quad ...(59)$$

[5] The safety stock of engine type *n* in product family *q* at the engine plant *m* in time period *t* is equal to the average daily demand of engine type *n* in product family *q* at the engine plant *m* multiplied by the number of days' worth of production specified by the policy at the engine plant *m*.

$$SS5_{nqmt} = AD5_{nqmt}STEM \qquad \forall n, q, m, t \quad ...(60)$$

The inventory of engine type *n* in product family *q* at the engine plant *m* in time period *t* should now be greater than or equal to the safety stock $SS5_{nqmt}$.

$$XIV5_{nqmt} \geq SS5_{nqmt} \qquad \forall n, q, m, t \quad ...(61)$$

[6] The safety stock of engine type *n* in product family *q* at the auto assembly plant *a* in time period *t* is equal to the average daily demand for engine type *n* in product family *q* at the auto assembly plant *a* multiplied by the number of days' worth of production specified by the policy at the auto assembly plant *a*.

$$SS6_{nqat} = AD6_{nqat}STEA \qquad \forall n, q, a, t \quad ...(62)$$

Now, the inventory of engine type *n* in product family *q* at the auto assembly plant *a* in time period *t* should be greater than or equal to the safety stock $SS6_{nqat}$.

$$XIV6_{nqat} \geq SS6_{nqat} \qquad \forall n, q, a, t \quad ...(63)$$

[7] The safety stock of vehicles with engine type *n* in product family *q* at the auto assembly plant *a* in time period *t* is equal to the average daily demand for vehicles with engine type *n* in product family *q* at the auto assembly plant *a* multiplied by the number of days' worth of production specified by the policy at the auto assembly plant *a*.

$$SS7_{nqat} = AD7_{nqat}STVA \qquad \forall n, q, a, t \quad ...(64)$$

The inventory of vehicles with engine type *n* in product family *q* at the auto assembly plant *a* in time period *t* should now be greater than or equal to the safety stock $SS7_{nqat}$.

$$XIV7_{nqat} \geq SS7_{nqat} \qquad \forall n, q, a, t \quad ...(65)$$

[8] Finally, the safety stock of rebuilt engines at the remanufacturing center *u* in time period *t* is equal to the average daily demand for rebuilt engines at the remanufacturing center *u*

multiplied by the number of days' worth of production specified by the policy at the remanufacturing center *u*.

$$SS8_{ut} = AD8_{ut}STEU \qquad \forall u, t \quad ...(66)$$

Now, the inventory of rebuilt engines at the remanufacturing center *u* in time period *t* should be greater than or equal to the safety stock $SS7_{ut}$.

$$XIV9_{ut} \geq SS8_{ut} \qquad \forall u, t \quad ...(67)$$

4.2.3.6 INTEGER RESTRICTIONS

$XTR1_{smkt}$, $XTR2_{makt}$, $XTR3_{adkt}$, $XTR4_{crkt}$, $XTR5_{mukt}$, $XTR6_{cukt}$, $XTR7_{udkt}$, $XIV3_{inst}$, $XIV4_{inmt}$, $XIV5_{nqmt}$, $XIV6_{nqat}$, $XIV7_{nqat}$, $XIV8_{ut}$, $XIV9_{ut}$, $XIT1_{sinmkt}$, $XIT2_{mnqakt}$, $XIT3_{anqdkt}$, $XIT4_{crkt}$, $XPS1_{sinmkt}$, $XPS2_{mnqakt}$, $XPS3_{anqdkt}$, $XPS4_{crkt}$, $XPS5_{miukt}$, $XPS6_{cukt}$, $XPS7_{udkt}$, XM_{nqmt}, $XM1_{inst}$, $XUS2_{inmt}$, $XUS3_{nqat}$, $XUS4_{ut}$, $SS3_{inst}$, $SS4_{inmt}$, $SS5_{nqmt}$, $SS6_{nqat}$, $SS7_{nqat}$, $SS8_{ut} = integer$

4.2.3.7 NON-NEGATIVITY RESTRICTIONS

XC_{st}, $XRLS_{st}$, $XRLM_{mt}$, $XRLU_{ut}$, $XOLS_{st}$, $XOLM_{mt}$, $XOLU_{ut}$, $XHLM_{mt}$, $XLLM_{mt}$, $XUS1_{st}$, $XTR1_{smkt}$, $XTR2_{makt}$, $XTR3_{adkt}$, $XTR4_{crkt}$, $XTR5_{mukt}$, $XTR6_{cukt}$, $XTR7_{udkt}$, $XIV1_{st}$, $XIV2_{st}$, $XIV3_{inst}$, $XIV4_{inmt}$, $XIV5_{nqmt}$, $XIV6_{nqat}$, $XIV7_{nqat}$, $XIV8_{ut}$, $XIV9_{ut}$, $XIT1_{sinmkt}$, $XIT2_{mnqakt}$, $XIT3_{anqdkt}$, $XIT4_{crkt}$, $XPS1_{sinmkt}$, $XPS2_{mnqakt}$, $XPS3_{anqdkt}$, $XPS4_{crkt}$, $XPS5_{miukt}$, $XPS6_{cukt}$, $XPS7_{udkt}$, XM_{nqmt}, $XM1_{inst}$, $XUS1_{st}$, $XUS2_{inmt}$, $XUS3_{nqat}$, $XUS4_{ut}$, $SS1_{st}$, $SS2_{st}$, $SS3_{inst}$, $SS4_{inmt}$, $SS5_{nqmt}$, $SS6_{nqat}$, $SS7_{nqat}$, $SS8_{ut} \geq 0$

4.3 Summary of the mathematical model

4.3.1 Objective function

$$Total\ costs = \sum_{s=1}^{S}\sum_{t=1}^{T}(P1 \times CP1 \times XC_{st} + P2 \times CP2 \times XC_{st})$$

$$+\sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR1_{smk}\, XTR1_{smkt} + \sum_{m=1}^{M}\sum_{a=1}^{A}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR2_{mak}\, XTR2_{makt}$$

$$+\sum_{a=1}^{A}\sum_{d=1}^{D}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR3_{adk}\, XTR3_{adkt} + \sum_{c=1}^{C}\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR4_{crk}\, XTR4_{crkt}$$

$$+\sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR5_{muk}\, XTR5_{mukt} + \sum_{c=1}^{C}\sum_{u=1}^{U}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR6_{cuk}\, XTR6_{cukt}$$

$$+\sum_{u=1}^{U}\sum_{d=1}^{D}\sum_{k=1}^{K}\sum_{t=1}^{T}CTR7_{udk}XTR7_{udkt}+\sum_{s=1}^{S}\sum_{t=1}^{T}CP1\times IVRS_{s}\times STRS\times XIV1_{st}$$

$$+\sum_{s=1}^{S}\sum_{t=1}^{T}CP2\times IVRS_{s}\times STRRS\times XIV2_{st}+\sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{s=1}^{S}\sum_{t=1}^{T}PP_{in}IVRS_{s}STPS\times XIV3_{inst}$$

$$+\sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{t=1}^{T}PP_{in}IVRM_{m}STPM\times XIV4_{inmt}$$

$$+\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{m=1}^{M}\sum_{t=1}^{T}PNE_{nq}IVRM_{m}STEM\times XIV5_{nqmt}$$

$$+\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{a=1}^{A}\sum_{t=1}^{T}PNE_{nq}IVRA_{a}STEA\times XIV6_{nqat}+\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{a=1}^{A}\sum_{t=1}^{T}PV_{nq}IVRA_{a}STVA\times XIV7_{nqat}$$

$$+\sum_{u=1}^{U}\sum_{t=1}^{T}PNRE\times IVRU_{u}STNEU\times XIV8_{ut}+\sum_{u=1}^{U}\sum_{t=1}^{T}PRE\times IVRU_{u}STEU\times XIV9_{ut}$$

$$+\sum_{s=1}^{S}\sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{t=1}^{T}PP_{in}IVRS_{s}LT1_{smk}XIT1_{\sin mkt}$$

$$+\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{a=1}^{A}\sum_{k=1}^{K}\sum_{t=1}^{T}PNE_{nq}IVRM_{m}LT2_{mak}XIT2_{mnqakt}$$

$$+\sum_{a=1}^{A}\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{d=1d}^{D}\sum_{k=1}^{K}\sum_{t=1}^{T}PV_{nq}IVRA_{a}LT3_{adk}XIT3_{anqdkt}$$

$$+\sum_{c=1}^{C}\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{t=1}^{T}PFH\times IVRC_{c}LT4_{crk}XIT4_{crkt}+\sum_{s=1}^{S}\sum_{t=1}^{T}CRLS_{st}XRLS_{st}$$

$$+\sum_{s=1}^{S}\sum_{t=1}^{T}COLS_{st}XOLS_{st}+\sum_{m=1}^{M}\sum_{t=1}^{T}CRLM_{mt}XRLM_{mt}+\sum_{m=1}^{M}\sum_{t=1}^{T}COLM_{mt}XOLM_{mt}$$

$$+\sum_{m=1}^{M}\sum_{t=1}^{T}CHLM_{mt}XHLM_{mt}+\sum_{m=1}^{M}\sum_{t=1}^{T}CLLM_{mt}XLLM_{mt}+\sum_{u=1}^{U}\sum_{t=1}^{T}CRLU_{ut}XRLU_{ut}$$

$$+\sum_{u=1}^{U}\sum_{t=1}^{T}COLU_{ut}XOLU_{ut}+\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{a=1}^{A}\sum_{k=1}^{K}\sum_{t=1}^{T}CH1_{nqa}XPS2_{mnqakt}+\sum_{c=1}^{C}\sum_{t=1}^{T}(ELV_{ct}+WRT)$$

$$-\sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{u=1}^{U}\sum_{k=1}^{K}\sum_{t=1}^{T}CP3_{miuk}XPS5_{miukt}$$

4.3.2 Constraints

$$\sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{K} XPS1_{sinmkt} \leq PC1_{st} \qquad \forall s,\ t\ldots(1)$$

$$\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{a=1}^{A}\sum_{k=1}^{K} XPS2_{mnqakt} \leq PC2_{mt} \qquad \forall m,\ t\ldots(2)$$

$$\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{d=1}^{D}\sum_{k=1}^{K} XPS3_{anqdkt} \leq PC3_{at} \qquad \forall a,\ t\ldots(3)$$

$$\sum_{r=1}^{R}\sum_{k=1}^{K} XPS4_{crkt} \leq PC4_{ct} \qquad \forall c,\ t\ldots(4)$$

$$\sum_{d=1}^{D}\sum_{k=1}^{K} XPS7_{udkt} \leq PC5_{ut} \qquad \forall u,\ t\ldots(5)$$

$$VP1 \times XIV1_{st} \leq FC1_{st} \qquad \forall s,\ t\ldots(6)$$

$$VP2 \times XIV2_{st} \leq FC2_{st} \qquad \forall s,\ t\ldots(7)$$

$$\sum_{i=1}^{I}\sum_{n=1}^{N} VP4_{in} XIV3_{inst} \leq \sum_{i=1}^{I}\sum_{n=1}^{N} FC3_{inst} \qquad \forall s,\ t\ldots(8)$$

$$\sum_{i=1}^{I}\sum_{n=1}^{N} VP4_{in} XIV4_{inmt} \leq \sum_{i=1}^{I}\sum_{n=1}^{N} FC4_{inmt} \qquad \forall m,\ t\ldots(9)$$

$$\sum_{n=1}^{N}\sum_{q=1}^{Q} VP3_{nq} XIV5_{nqmt} \leq \sum_{n=1}^{N}\sum_{q=1}^{Q} FC5_{nqmt} \qquad \forall m,\ t\ldots(10)$$

$$\sum_{n=1}^{N}\sum_{q=1}^{Q} VP3_{nq} XIV6_{nqat} \leq \sum_{n=1}^{N}\sum_{q=1}^{Q} FC6_{nqat} \qquad \forall a,\ t\ldots(11)$$

$$\sum_{n=1}^{N}\sum_{q=1}^{Q} VP5XIV7_{nqat} \leq \sum_{n=1}^{N}\sum_{q=1}^{Q} FC7_{nqat} \qquad \forall a,\ t\ldots(12)$$

$$VP7 \times XIV8_{ut} \leq FC8_{ut} \qquad \forall u,\ t\ldots(13)$$

$$VP7 \times XIV9_{ut} \leq FC9_{ut} \qquad \forall u,\ t\ldots(14)$$

$$XRLS_{st} \leq MLHS_{st} \qquad \forall s,\ t\ldots(15)$$

$$XOLS_{st} \leq fs_s XRLS_{st} \qquad \forall s,\ t\ldots(16)$$

$$XRLS_{st} + XOLS_{st} \geq \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{K} RHS_{ins} XPS1_{sinmkt} \qquad \forall s,\ t\ldots(17)$$

$$XRLM_{mt} \leq MLHM_{mt} \qquad \forall m,\ t\ldots(18)$$

$$XOLM_{mt} \leq fm_m XRLM_{mt} \qquad \forall m,\ t\ldots(19)$$

$$XRLM_{mt} + XOLM_{mt} + XHLM_{mt} - XLLM_{mt} \geq \sum_{n=1}^{N}\sum_{q=1}^{Q} RHM_{nqm} XM_{nqmt} \qquad \forall m, t \ldots (20)$$

$$XRLU_{ut} \leq MLHU_{ut} \qquad \forall u, t \ldots (21)$$

$$XOLU_{ut} \leq fu_u XRLU_{ut} \qquad \forall u, t \ldots (22)$$

$$XRLU_{ut} + XOLU_{ut} \geq \sum_{c=1}^{C}\sum_{k=1}^{K} RHU_u XPS6_{cukt} \qquad \forall u, t \ldots (23)$$

$$XTR1_{smkt} \geq \max\left(\frac{\sum_{i=1}^{I}\sum_{n=1}^{N} WP3_{in} XPS1_{\sin mkt}}{WC_k}, \frac{\sum_{i=1}^{I}\sum_{n=1}^{N} VP4_{in} XPS1_{\sin mkt}}{VC_k} \right) \qquad \forall s, m, k, t \ldots (24)$$

$$XTR2_{makt} \geq \frac{\sum_{n=1}^{N}\sum_{q=1}^{Q} XPS2_{mnqakt}}{\sum_{n=1}^{N}\sum_{q=1}^{Q} NN_{nqk}} \qquad \forall m, a, k, t \ldots (25)$$

$$XTR3_{adkt} \geq \frac{\sum_{n=1}^{N}\sum_{q=1}^{Q} XPS3_{anqdkt}}{NNN_k} \qquad \forall a, d, k, t \ldots (26)$$

$$XTR4_{crkt} \geq \max\left(\frac{WP4 \times XPS4_{crkt}}{WC_k}, \frac{VP6 \times XPS4_{crkt}}{VC_k} \right) \qquad \forall c, r, k, t \ldots (27)$$

$$\sum_{t=1}^{T} XTR5_{mukt} \geq \max\left(\frac{\sum_{i=1}^{I}\sum_{t=1}^{T} WP1 \times XPS5_{miukt}}{WC_k}, \frac{\sum_{i=1}^{I}\sum_{t=1}^{T} VP8 \times XPS5_{miukt}}{VC_k} \right) \qquad \forall m, i, u, k \ldots (28)$$

$$\sum_{t=1}^{T} XTR6_{cukt} \geq \max\left(\frac{\sum_{t=1}^{T} WP2 \times XPS6_{cukt}}{WC_k}, \frac{\sum_{t=1}^{T} VP7 \times XPS6_{cukt}}{VC_k} \right) \qquad \forall c, u, k, t \ldots (29)$$

$$\sum_{t=1}^{T} XTR7_{udkt} \geq \max\left(\frac{\sum_{t=1}^{T} WP2 \times XPS7_{udkt}}{WC_k}, \frac{\sum_{t=1}^{T} VP7 \times XPS7_{udkt}}{VC_k} \right) \qquad \forall u, d, k, t \ldots (30)$$

$$XIV1_{st} + XIV2_{st} = XIV1_{s(t-1)} + XIV2_{s(t-1)} + XC_{st} - XUS1_{st} \qquad \forall s, t \ldots (31)$$

$$XIV3_{inst} = XIV3_{ins(t-1)} + XM1_{inst} - \sum_{m=1}^{M}\sum_{k=1}^{K} XPS1_{sin\,mkt} \qquad \forall\, i,\ n,\ s,\ t \ldots(32)$$

$$\sum_{i=1}^{I}\sum_{n=1}^{N} WP3_{in}\, XM1_{inst} = 0.98 XUS1_{st} \qquad \forall\, s,\ t \ldots(33)$$

$$\sum_{s=1}^{S}\sum_{k=1}^{K} XPS1_{sin\,mkt} \geq AD4_{inmt} \qquad \forall\, i,\ n,\ m,\ t \ldots(34)$$

$$XIV4_{inmt} = XIV4_{inm(t-1)} + \sum_{s=1}^{S}\sum_{k=1}^{K} XPS1_{sin\,mk(t-LT1_{smk})} - \sum_{u=1}^{U}\sum_{k=1}^{K} XPS5_{miukt} - XUS2_{inmt}$$

$$\forall\, i,\ n,\ m,\ t \ \ldots(35)$$

$$XIV5_{nqmt} = XIV5_{nqm(t-1)} + XM_{nqmt} - \sum_{a=1}^{A}\sum_{k=1}^{K} XPS2_{mnqakt} - WRT \qquad \forall\, n,\ q,\ m,\ t \ldots(36)$$

$$XUS2_{inmt} = BOM_{in} XM_{nqmt} \qquad \forall\, i,\ n,\ q,\ m,\ t \ldots(37)$$

$$\sum_{m=1}^{M}\sum_{k=1}^{K} XPS2_{mnqakt} \geq AD6_{nqat} \qquad \forall\, n,\ q,\ m,\ t \ldots(38)$$

$$XIV6_{nqat} = XIV6_{nqa(t-1)} + \sum_{m=1}^{M}\sum_{k=1}^{K} XPS2_{mnqak(t-LT2_{mak})} - XUS3_{nqat} \qquad \forall\, n,\ q,\ a,\ t \ldots(39)$$

$$XIV7_{nqat} = XIV7_{nqa(t-1)} + XUS3_{nqat} - \sum_{d=1}^{D}\sum_{k=1}^{K} XPS3_{anqdkt} \qquad \forall\, n,\ q,\ a,\ t \ldots(40)$$

$$\sum_{a=1}^{A}\sum_{k=1}^{K} XPS3_{anqdkt} \geq AD9_{nqat} \qquad \forall\, n,\ q,\ d,\ t \ldots(41)$$

$$\sum_{r=1}^{R}\sum_{k=1}^{K} XPS4_{crkt} = ELV_{ct} + WRT \qquad \forall\, c,\ t \ \ldots(42)$$

$$\sum_{u=1}^{U}\sum_{k=1}^{K} XPS6_{cukt} = 18\%(ELV_{ct} + WRT) \qquad \forall\, c,\ t \ldots(43)$$

$$XIV8_{ut} = XIV8_{u(t-1)} + \sum_{c=1}^{C}\sum_{k=1}^{K} XPS6_{cuk(t-LT6_{cuk})} - XUS4_{ut} \qquad \forall\, u,\ t \ldots(44)$$

$$XIV9_{ut} = XIV9_{u(t-1)} + XUS4_{ut} - \sum_{d=1}^{D}\sum_{k=1}^{K} XPS7_{udkt} \qquad \forall\, u,\ t \ldots(45)$$

$$\sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{k=1}^{K} XPS5_{miukt} = \sum_{d=1}^{D}\sum_{k=1}^{K} XPS7_{udkt} \qquad \forall\, u,\ t \ldots(46)$$

$$\sum_{u=1}^{U}\sum_{k=1}^{K} XPS7_{udkt} \geq AD10_{dt} \qquad \forall\, d,\ t \ldots(47)$$

$$XIT1_{sinmkt} = XIT1_{sinmk(t-1)} + XPS1_{sinmkt} - XPS1_{sinmk(t-LT1_{smk})} \qquad \forall\, s,\ i,\ n,\ m,\ k,\ t \ldots(48)$$

$$XIT2_{mnqakt} = XIT2_{mnqak(t-1)} + XPS2_{mnqakt} - XPS2_{mnqak(t-LT2_{mak})} \qquad \forall\, m,\ n,\ q,\ a,\ k,\ t \ldots(49)$$

$$XIT3_{anqdkt} = XIT3_{anqdk(t-1)} + XPS3_{anqdkt} - XPS3_{anqdk(t-LT3_{adk})} \qquad \forall\, a,\ n,\ q,\ d,\ k,\ t \ldots(50)$$

$$XIT4_{crkt} = XIT4_{crk(t-1)} + XPS4_{crkt} - XPS4_{crk(t-LT4_{crk})} \qquad \forall\, c,\ r,\ k,\ t \ldots(51)$$

$$SS1_{st} = AD1_{st}STRS \qquad \forall\, s,\ t \ldots(52)$$

$$XIV1_{st} \geq SS1_{st} \qquad \forall\, s,\ t \ldots(53)$$

$$SS2_{st} = AD2_{st}STRRS \qquad \forall\, s,\ t \ldots(54)$$

$$XIV2_{st} \geq SS2_{st} \qquad \forall\, s,\ t \ldots(55)$$

$$SS3_{inst} = AD3_{inst}STPS \qquad \forall\, i,\ n,\ s,\ t \ldots(56)$$

$$XIV3_{inst} \geq SS3_{inst} \qquad \forall\, i,\ n,\ s,\ t \ldots(57)$$

$$SS4_{inmt} = AD4_{inmt}STPM \qquad \forall\, i,\ n,\ m,\ t \ldots(58)$$

$$XIV4_{inmt} \geq SS4_{inmt} \qquad \forall\, i,\ n,\ m,\ t \ldots(59)$$

$$SS5_{nqmt} = AD5_{nqmt}STEM \qquad \forall\, n,\ q,\ m,\ t \ldots(60)$$

$$XIV5_{nqmt} \geq SS5_{nqmt} \qquad \forall\, n,\ q,\ m,\ t \ldots(61)$$

$$SS6_{nqat} = AD6_{nqat}STEA \qquad \forall\, n,\ q,\ a,\ t \ldots(62)$$

$$XIV6_{nqat} \geq SS6_{nqat} \qquad \forall\, n,\ q,\ a,\ t \ldots(63)$$

$$SS7_{nqat} = AD7_{nqat}STVA \qquad \forall\, n,\ q,\ a,\ t \ldots(64)$$

$$XIV7_{nqat} \geq SS7_{nqat} \qquad \forall\, n,\ q,\ a,\ t \ldots(65)$$

$$SS8_{ut} = AD8_{ut}STEU \qquad \forall\, u,\ t \ldots(66)$$

$$XIV9_{ut} \geq SS8_{ut} \qquad \forall\, u,\ t \ldots(67)$$

$XTR1_{smkt}$, $XTR2_{makt}$, $XTR3_{adkt}$, $XTR4_{crkt}$, $XTR5_{mukt}$, $XTR6_{cukt}$, $XTR7_{udkt}$, $XIV3_{inst}$, $XIV4_{inmt}$, $XIV5_{nqmt}$, $XIV6_{nqat}$, $XIV7_{nqat}$, $XIV8_{ut}$, $XIV9_{ut}$, $XIT1_{sinmkt}$, $XIT2_{mnqakt}$, $XIT3_{anqdkt}$, $XIT4_{crkt}$, $XPS1_{sinmkt}$, $XPS2_{mnqakt}$, $XPS3_{anqdkt}$, $XPS4_{crkt}$, $XPS5_{miukt}$, $XPS6_{cukt}$, $XPS7_{udkt}$, XM_{nqmt}, $XM1_{inst}$, $XUS2_{inmt}$, $XUS3_{nqat}$, $XUS4_{ut}$, $SS3_{inst}$, $SS4_{inmt}$, $SS5_{nqmt}$, $SS6_{nqat}$, $SS7_{nqat}$, $SS8_{ut}$= integer

XC_{st}, $XRLS_{st}$, $XRLM_{mt}$, $XRLU_{ut}$, $XOLS_{st}$, $XOLM_{mt}$, $XOLU_{ut}$, $XHLM_{mt}$, $XLLM_{mt}$, $XUS1_{st}$, $XTR1_{smkt}$, $XTR2_{makt}$, $XTR3_{adkt}$, $XTR4_{crkt}$, $XTR5_{mukt}$, $XTR6_{cukt}$, $XTR7_{udkt}$, $XIV1_{st}$, $XIV2_{st}$, $XIV3_{inst}$, $XIV4_{inmt}$, $XIV5_{nqmt}$, $XIV6_{nqat}$, $XIV7_{nqat}$, $XIV8_{ut}$, $XIV9_{ut}$, $XIT1_{sinmkt}$, $XIT2_{mnqakt}$, $XIT3_{anqdkt}$, $XIT4_{crkt}$,

$XPS1_{sinmkt}$, $XPS2_{mnqakt}$, $XPS3_{anqdkt}$, $XPS4_{crkt}$, $XPS5_{miukt}$, $XPS6_{cukt}$, $XPS7_{udkt}$, XM_{nqmt}, $XM1_{inst}$, $XUS1_{st}$, $XUS2_{inmt}$, $XUS3_{nqat}$, $XUS4_{ut}$, $SS1_{st}$, $SS2_{st}$, $SS3_{inst}$, $SS4_{inmt}$, $SS5_{nqmt}$, $SS6_{nqat}$, $SS7_{nqat}$, $SS8_{ut} \geq 0$

CHAPTER 5 RESULTS

This chapter uses real-life cases from Ford's U.S. engine operations to verify the proposed model and generate an optimal solution with LINGO.

The following section presents the proposed model's input parameters and data.

5.1 Parameters

5.1.1 Parameters for transportation

Table 5.1 shows the purchasing cost of aluminum ingots and recycled aluminum, and the proportion of each in the input mix at the aluminum casting plant.

Table 5.2 shows the transportation costs and the transportation lead times from the aluminum casting plant to engine plants using different transport modes. The aluminum casting plant only sends products to engine plants 2, 3 and 5. For example, from aluminum casting plant s=1 to engine plant m=2 using transport mode k=1, the transportation cost is $650 per full truck load and the transportation lead time is 3 days.

Table 5.3 shows the transportation costs and the transportation lead times from engine plants to auto assembly plants using different transport modes. For example, from engine plant m=1 to auto assembly plant a=8, using transport mode k=1, the transportation cost is $650 per full truck load and the transportation lead time is 5 days. A lead time of zero means that shipments made during the day are generally received in the same day. The lead times are rounded up to the nearest integer value.

Table 5.4 shows the transportation costs and the transportation lead times from auto assembly plants to dealerships using different transport modes. For example, from auto assembly plant a=1 to dealership d=1, using transport mode k=1, the transportation cost is $125 per full truck load and the transportation lead time is 3 days.

Table 5.5 shows the transportation costs and the transportation lead times from collection centers to recycling centers using different transport modes. For example, from collection center c=1 to recycling center r=1, using transport mode k=1, the transportation cost is $350 per full truck load and the transportation lead time is 1 day.

Table 5.6 shows the transportation costs and the transportation lead times from engine plants to remanufacturing centers using different transport modes. For example, from engine plant m=1 to remanufacturing center u=1, using transport mode k=1, the transportation cost is $250 per full truck load, the transportation lead time is 2 days and the price of engine parts is $70 per unit.

Table 5.7 shows the transportation costs and the transportation lead times from collection centers to remanufacturing centers using different transport modes. For example, from collection center c=1 to remanufacturing center u=1 using transport mode k=1, the transportation cost is $210 per full truck load and the transportation lead time is 1 day.

Table 5.8 shows the transportation costs and the transportation lead times from remanufacturing centers to dealerships using different transport modes. For example, from remanufacturing center u=1 to dealership d=1 using transport mode k=1, the transportation cost is $560 per full truck load and the transportation lead time is 1 day.

Purchasing cost ($/lb)		Percentage in input mix	
aluminum ingots *CP1*	recycled aluminum *CP2*	aluminum ingots *P1*	recycled aluminum *P2*
1.15	0.85	15%	85%

Table 5.1 Aluminum purchasing cost

Aluminum casting plant, *s*	Engine plant, *m*	Transport mode, *k*	Transportation cost, $CTR1_{smk}$ ($/FTL)	Lead time, $LT1_{smk}$ (days)
1	2	1	$650.00	3
1	2	2	$3,500.00	1
1	3	1	$650.00	5
1	3	2	$3,500.00	2
1	5	1	$650.00	5
1	5	2	$3,500.00	1

Table 5.2 Transportation cost and transportation lead time from the aluminum casting plant to engine plants

5.1.2 Labor, handling and inventory related parameters

Tables 5.9 through 5.11 show the inventory carrying cost rate and the ratio of overtime to the regular-time labor hours used to calculate labor costs at the aluminum casting plant, engine plants and remanufacturing centers.

Table 5.12 and Table 5.13 show, respectively, the inventory carrying cost rate and the handling cost at the auto assembly plants. For example, the handling cost for engine type n=4 in product family q=1 handled at auto assembly plant a=1 is $1,511 per unit.

Table 5.14 shows the cost of handling end-of-life vehicles and the inventory carrying cost rate at the collection centers.

Table 5.15 shows the regular-time labor costs, overtime labor costs and the value of the maximum allowable limit — based on the number of employees working at this plant, multiplied by eight hours per day — at the aluminum casting plant.

Table 5.16 shows the production rate at the aluminum casting plant. For example, the production rate of engine part i=1 for engine type n=3 produced at the aluminum casting plant s=1 is 0.03 hours.

Table 5.17 shows the regular-time labor cost, overtime labor cost, hiring cost, layoff cost and the maximum allowable limit on labor hours at the engine plants.

Table 5.18 shows the hours-per-engine requirements at the engine plants. For example, the requirement of engine type n=1 in product family q=1 produced at engine plant m=1 is 4.53 hours per engine.

Table 5.19 shows the regular-time labor costs, overtime labor costs, and the maximum allowable limit on labor hours at the remanufacturing centers.

Table 5.20 shows the hours-per-engine requirements at the remanufacturing centers. For example, the requirement is 8 hours per engine at either remanufacturing center.

5.1.3 Parameters for volume capacity, weight capacity and price of rebuildable, rebuilt engines and flattened hulks

Table 5.21 shows the per unit volume of aluminum ingots, recycled aluminum and the average volume of an engine, engine part i=3 and a unit of flattened hulk. It also shows the average area a vehicle occupies.

Engine plants, *m*	Auto assembly plants, *a*	Transport mode, *k*	Transportation cost, $CTR2_{mak}$ ($/FTL)	Lead time, $LT2_{mak}$ (days)
1	8	1	$650.00	5
1	8	2	$3,500.00	0
1	12	1	$250.00	1
1	12	2	$3,500.00	0
2	1	1	$650.00	5
2	1	2	$3,500.00	1
2	4	1	$650.00	3
2	4	2	$3,500.00	0
2	9	1	$650.00	8
2	9	2	$5,200.00	0
2	10	1	$650.00	8
2	10	2	$5,200.00	0
3	2	1	$250.00	1
3	2	2	$3,500.00	0
4	11	1	$650.00	1
4	11	2	$3,500.00	0
5	1	1	$250.00	1
5	1	2	$3,500.00	0
5	3	1	$250.00	1
5	3	2	$3,500.00	0
5	4	1	$650.00	2
5	4	2	$3,500.00	0
5	5	1	$650.00	5
5	5	2	$3,500.00	0
5	6	1	$650.00	3
5	6	2	$3,500.00	1
5	7	1	$650.00	2
5	7	2	$3,500.00	0
5	13	1	$250.00	1
5	13	2	$3,500.00	0

Table 5.3 Transportation cost and transportation lead time from engine plants to auto assembly plants

Auto assembly plant, a	Dealership, d	Transport mode, k	Transportation cost, $CTR3_{adk}$ ($/FTL)	Lead time, $LT3_{adk}$ (days)
1	1	1	$125.00	3
1	1	2	$3,500.00	1
2	1	1	$250.00	2
2	1	2	$3,500.00	1
3	1	1	$650.00	10
3	1	2	$3,500.00	2
4	1	1	$650.00	1
4	1	2	$3,500.00	1
5	1	1	$650.00	1
5	1	2	$3,500.00	1
6	1	1	$250.00	0
6	1	2	$3,500.00	1
7	1	1	$125.00	5
7	1	2	$3,500.00	2
8	1	1	$250.00	1
8	1	2	$3,500.00	0
9	1	1	$650.00	3
9	1	2	$3,500.00	1
10	1	1	$250.00	4
10	1	2	$3,500.00	0
11	1	1	$650.00	0
11	1	2	$3,500.00	0
12	1	1	$250.00	0
12	1	2	$3,500.00	0
13	1	1	$650.00	3
13	1	2	$3,500.00	1

Table 5.4 Transportation cost and transportation lead time from auto assembly plants to dealerships

Table 5.22 shows the volume and weight of engine part i=1 and the per unit price of the engine part. The bill of material rate is the number of units of engine part i used per item of engine type n.

Table 5.23 shows the average weight of engine part i=3, an engine and a flattened hulk. Table 5.24 shows the average price of a rebuildable engine, a rebuilt engine and a flattened hulk.

Table 5.25 shows the engine volume and price data. For example, the volume of engine type n=1 in product family q=1 is 25 cubic feet and the price is $2,595. The table also shows the average price of a vehicle with engine type n in product family q.

Table 5.26 shows the transport mode's weight and volume capacity and the number of vehicles per FTL for each transport mode. For example, 10 vehicles can be loaded on a FTL using transport mode k=1.

Table 5.27 shows the number of engines per full truck load for each transport mode. For example, 80 units of engine type n=1 in product family q=1 can be loaded in a FTL using transport mode k=1.

Collection center, c	Recycling center, r	Transport mode, k	Transportation cost, $CTR4_{crk}$ ($/FTL)	Lead time, $LT4_{crk}$ (days)
1	1	1	$350.00	1
1	1	2	$301.00	2
1	2	1	$311.50	1
1	2	2	$315.00	1
2	1	1	$318.50	2
2	1	2	$301.00	1
2	2	1	$241.50	2
2	2	2	$280.00	1
3	1	1	$325.50	1
3	1	2	$346.50	2
3	2	1	$210.00	1
3	2	2	$280.00	2

Table 5.5 Transportation cost and transportation lead time from collection centers to recycling centers

5.1.4 Parameters related to safety stocks, average demand and production capacity and storage space

Table 5.28 shows the level of safety stock kept at the aluminum casting plant, engine plants, auto assembly plants and remanufacturing centers for different products.

For example, the level of safety stock for aluminum ingots held at the aluminum casting plant is 2 days worth of production.

Tables 5.29 through 5.39 display the average demand for products at different stages of the chain, and the corresponding production capacity and the available storage space.

Engine plant, *m*	Remanufacturing center, *u*	Transport mode, *k*	Transportation cost, $CTR5_{muk}$ ($/FTL)	Lead time, $LT5_{muk}$ (days)	The price of engine parts *i*=3, $CP3_{miuk}$ ($/unit)
1	1	1	$250.00	2	$70.00
1	1	2	$3,500.00	0	$90.00
2	1	1	$250.00	2	$70.00
2	1	2	$3,500.00	0	$90.00
3	1	1	$250.00	3	$70.00
3	1	2	$3,500.00	1	$90.00
4	1	1	$250.00	3	$70.00
4	1	2	$3,500.00	2	$90.00
5	1	1	$250.00	2	$70.00
5	1	2	$3,500.00	0	$90.00
1	2	1	$250.00	5	$70.00
1	2	2	$3,500.00	1	$90.00
2	2	1	$250.00	2	$70.00
2	2	2	$3,500.00	0	$90.00
3	2	1	$250.00	3	$70.00
3	2	2	$3,500.00	1	$90.00
4	2	1	$250.00	5	$70.00
4	2	2	$3,500.00	1	$90.00
5	2	1	$250.00	3	$70.00
5	2	2	$3,500.00	0	$90.00

Table 5.6 Transportation cost and transportation lead time from engine plants to remanufacturing centers and the average price of engine parts sold by engine plants to remanufacturing centers

Collection center, c	Remanufacturing center, u	Transport mode, k	Transportation cost, $CTR6_{cuk}$ ($/FTL)	Lead time, $LT6_{cuk}$ (days)
1	1	1	$210.00	1
1	1	2	$290.00	3
2	1	1	$142.00	1
2	1	2	$120.00	2
3	1	1	$109.00	1
3	1	2	$99.00	0
1	2	1	$215.00	2
1	2	2	$295.00	0
2	2	1	$147.00	2
2	2	2	$125.00	1
3	2	1	$117.00	2
3	2	2	$109.00	1

Table 5.7 Transportation cost and transportation lead time from collection centers to remanufacturing centers

Remanufacturing center, u	Dealership, d	Transport mode, k	Transportation cost, $CTR7_{udk}$ ($/FTL)	Lead time, $LT7_{udk}$ (days)
1	1	1	$560.00	1
1	1	2	$3,500.00	0
2	1	1	$560.00	1
2	1	2	$3,500.00	2

Table 5.8 Transportation cost and transportation lead time from remanufacturing centers to dealerships

Aluminum casting plant, s	Inventory carrying cost rate, $IVRS_s$ ($/day)	Ratio of overtime to regular-time labor hours, fs_s
1	0.0003	0.2

Table 5.9 Inventory carrying cost rate and ratio of overtime to regular-time labor hours at the aluminum casting plant

Engine plant, m	Inventory carrying cost rate, $IVRM_m$ ($/day)	Ratio of overtime to regular-time labor hours, fm_m
1	0.0003	0.2
2	0.0003	0.2
3	0.0003	0.2
4	0.0003	0.2
5	0.0003	0.2

Table 5.10 Inventory carrying cost rate and ratio of overtime to regular-time labor hours at engine plants

Remanufacturing center, u	Inventory carrying cost rate, $IVRU_u$ ($/day)	Ratio of overtime to regular-time labor hours, fu_u
1	0.0003	0.2
2	0.0003	0.2

Table 5.11 Inventory carrying cost rate and ratio of overtime to regular-time labor hours at remanufacturing centers

Auto assembly plant, a	Inventory carrying cost rate, $IVRA_a$ ($/day)
1	0.0003
2	0.0003
3	0.0003
4	0.0003
5	0.0003
6	0.0003
7	0.0003
8	0.0003
9	0.0003
10	0.0003
11	0.0003
12	0.0003
13	0.0003

Table 5.12 Inventory carrying cost rate at auto assembly plants

Auto assembly plant, *a*	Engine type, *n*	Product family, *q*	Handling cost, $CH1_{nqa}$ (\$/unit)
1	4	1	$1,511.10
1	6	1	$1,511.10
1	7	2	$1,511.10
2	4	1	$1,246.30
3	6	2	$1,511.10
4	4	1	$1,506.50
4	6	2	$1,506.50
5	6	2	$1,544.00
6	6	2	$1,283.80
7	6	2	$1,511.10
8	2	1	$1,367.90
9	4	1	$1,445.40
10	3	1	$1,445.40
10	4	1	$1,445.40
11	5	1	$1,445.40
12	1	1	$1,383.00
13	6	2	$1,511.10

Table 5.13 Handling costs at auto assembly plants

Collection center, *c*	Handling cost, $CH2_c$ (\$/unit)	Inventory carrying cost rate, $IVRC_c$
1	$280.00	0.0003
2	$329.00	0.0003
3	$320.00	0.0003

Table 5.14 Handling cost and inventory carrying cost rate at collection centers

Aluminum Casting Plant, *s*	Period, *t*	Regular-time labor cost, $CRLS_{st}$ (\$/hour)[1]	Overtime labor cost, $COLS_{st}$ (\$/hour)	Max. allowable labor, $MLHS_{st}$ (hrs/day)[2]
1	1	$67.10	$75.00	3,600

Table 5.15 Labor cost and maximum allowable labor hours at the aluminum casting plant

[1] Information on labor cost is available at

http://www.ford.com/en/company/about/sustainability/report/relData.htm#B

http://www.bls.gov/news.release/pdf/prod2.pdf

[2] Information on the number of employees working at the aluminum casting plant is available at

http://media.ford.com/facilities/plant_display.cfm?plant_id=138

Aluminum casting plant, s	Engine part, i	Engine type, n	Production rate, RHS_{ins} (hrs/part)
1	1	3	0.03
1	1	4	0.03
1	1	6	0.03

Table 5.16 Production rate at the aluminum casting plant

Engine Plants, m	Period, t	Regular-time labor cost, $CRLM_{mt}$ ($/hour)	Overtime labor cost, $COLM_{mt}$ ($/hour)	Hiring cost, $CHLM_{mt}$ ($/hour)	Layoff cost, $CLLM_{mt}$ ($/hour)	Max. allowable labor, $MLHM_{mt}$ (hrs/month)[3]
1	1	$67.10	$75.00	$80.00	$160.00	6,784
2	1	$67.10	$75.00	$80.00	$160.00	13,240
3	1	$67.10	$75.00	$80.00	$160.00	13240
4	1	$67.10	$75.00	$80.00	$160.00	7,120
5	1	$67.10	$75.00	$80.00	$160.00	9,400

Table 5.17 Labor cost and maximum allowable labor hours at engine plants

Engine type, n	Product family, q	Engine plant, m	Hours per engine, RHM_{nqm} (hrs/engine)[4]
1	1	1	4.53
2	1	1	4.53
3	1	2	5.99
4	1	2	5.99
4	1	3	4.77
5	1	4	3.98
6	1	5	4.2
6	2	5	4.2
7	2	5	4.2

Table 5.18 Hours per engine at engine plants

[3] Information on labor cost is available at http://media.ford.com/facilities/plant_display.cfm?plant_id=83
http://media.ford.com/facilities/plant_display.cfm?plant_id=31
http://media.ford.com/facilities/plant_display.cfm?plant_id=30
http://media.ford.com/facilities/plant_display.cfm?plant_id=26
http://media.ford.com/facilities/plant_display.cfm?plant_id=40

[4] The Harbour Report 2005

Remanufacturing center, u	Period, t	Regular-time labor cost, $CRLU_{ut}$ (\$/hour)	Overtime labor cost, $COLU_{ut}$ (\$/hour)	Max. allowable labor, $MLHU_{ut}$ (hours)
1	1	$67.10	$75.00	280,000
2	1	$67.10	$75.00	150,000

Table 5.19 Labor cost and maximum allowable labor hours at remanufacturing centers

Remanufacturing center, u	Hours per engine, RHU_u (hrs/engine) [5]
1	8
2	8

Table 5.20 Production rate at remanufacturing centers

Per unit volume of aluminum ingots $VP1$ (cu.ft/lb)	Per unit volume of recycled aluminum $VP2$ (cu.ft/lb)	Per unit area of vehicles $VP5$ (sq.ft/unit)	Per unit volume of flattened hulks $VP6$ (cu.ft/unit)	Per unit volume of engines $VP7$ (cu.ft/unit)	Per unit volume of engine part i=3, $VP8$ (cu.ft/unit)
0.0059	0.0059	100	125	36	0.409

Table 5.21 The volume and area data

Part, i	Engine type, n	Bill of material rate, BOM_{in}	Per unit volume of engine part, $VP4_{in}$ (cu.ft/unit)	Per unit weight of engine part, $WP3_{in}$ (lbs/unit)	Per unit price of engine part, PP_{in} (\$/unit)
1	3	1	3.1	180	$1,100.00
1	4	1	3.5	200	$1,190.00
1	6	1	3.9	210	$1,202.00

Table 5.22 Bill of material rate, volume, weight and price of one unit of engine part i=1

Average weight of engine part i=3, $WP1$ (lbs)	Average weight of an engine, $WP2$ (lbs)	Average weight of a flattenedhulk, $WP4$ (lbs)
11	500	2,000

Table 5.23 Average weight of an engine part, engine and flattened hulk

Average price of a rebuildable engine $PNRE$ (\$/unit)	Average price of a rebuilt engine PRE (\$/unit)	Average price of a flattened hulk PFH (\$/unit)
500	2,000	500

Table 5.24 Average price of a rebuildable engine, the rebuilt engine and the flattened hulk

[5] Hindo, B., 2006, Happiness is a worn part. BusinessWeek (September 25).

Engine type, n	Product family, q	Volume of an engine, $VP3_{nq}$ (cu.ft/unit)	Price of an engine, PNE_{nq} ($/unit)	Average price of a vehicle, PV_{nq} ($/unit)
1	1	25	$2,595.00	$19,462.50
2	1	25	$2,595.00	$19,462.50
3	1	26	$3,800.00	$28,500.00
4	1	28	$3,800.00	$28,500.00
5	1	28	$4,980.00	$37,350.00
6	1	30	$6,849.00	$51,367.50
6	2	33	$6,849.00	$51,367.50
7	2	33	$4,200.00	$31,500.00

Table 5.25 Engine volume and price and the average price of a vehicle with engine type n

Mode, k	Weight capacity of transportation, WC_k (lbs)	Volume capacity of transportation, VC_k (cu.ft)	Number of vehicles per FTL, NNN_k (units)
1	100,000	1,000	10
2	1,000,000	8,000	20

Table 5.26 Weight and volume capacities and the number of vehicles per FTL with respect to each transport mode

Engine type, n	Product family, q	Mode, k	Number of engines per FTL, NN_{nqk} (units)
1	1	1	80
2	1	1	80
3	1	1	80
4	1	1	80
5	1	1	80
6	1	1	45
6	2	1	45
7	2	1	45
1	1	2	240
2	1	2	240
3	1	2	240
4	1	2	240
5	1	2	240
6	1	2	144
6	2	2	144
7	2	2	144

Table 5.27 The number of engines per FTL with respect to each transport mode

Levels of safety stock (days)							
At the aluminum casting plant			At engine plants		At auto assembly plants		At re-mfg centers
Aluminum ingots *STRS*	Recycled aluminum *STRRS*	Engine parts *STPS*	Engine parts *STPM*	Engines *STEM*	Engines *STEA*	Vehicles *STVA*	Engines rebuilt *STEU*
2	2	2	2	2	2	2	2

Table 5.28 Levels of safety stock

Aluminum casting plant, s	Period, T	Average demand for aluminum ingots, $AD1_{st}$ (lbs/day)	Average demand for recycled aluminum, $AD2_{st}$ (lbs/day)	Production capacity, $PC1_{st}$ (parts/day)	Storage space for aluminum ingots, $FC1_{st}$ (cu.ft.)	Storage space for aluminum ingots, $FC1_{st}$ (cu.ft.)
1	1	94,813.5	537,276.5	500,000	50,000	50,000

Table 5.29 Average demand and storage space for aluminum ingots and recycled aluminum and production capacity at the aluminum casting plant

Engine part, i	Engine type, n	Aluminum casting plant, s	Period, t	Average demand for engine parts, $AD3_{inst}$ (units/day)	Storage space for engine parts, $FC3_{inst}$ (cu.ft./unit)
1	3	1	1	189	30,000
1	4	1	1	2,857	30,000
1	6	1	1	127	30,000

Table 5.30 Average demand and storage space for engine parts at the aluminum casting plant

Engine part, i	Engine type, n	Engine plants, m	Period, t	Average demand for engine parts, $AD4_{inmt}$ (units/day)	Storage space for engine parts, $FC4_{inmt}$ (cu.ft./unit)
1	3	2	1	189	30,000
1	4	2	1	2,130	30,000
1	4	3	1	727	30,000
1	6	5	1	127	30,000

Table 5.31 Average demand and storage space for engine parts at engine plants

Engine type, n	Product family, q	Engine plant, m	Period, t	Average demand for engines $AD5_{nqmt}$ (units/day)	Storage space for engines, $FC5_{nqmt}$ (cu.ft./unit)
1	1	1	1	561	300,000
2	1	1	1	325	300,000
3	1	2	1	189	300,000
4	1	2	1	2,130	300,000
4	1	3	1	727	300,000
5	1	4	1	562	300,000
6	1	5	1	127	300,000
6	2	5	1	2,465	300,000
7	2	5	1	127	300,000

Table 5.32 Average demand and storage space for engines at engine plants

Engine plant, m	Period, t	Production capacity, $PC2_{mt}$ (units/day)
1	1	300,000
2	1	300,000
3	1	300,000
4	1	300,000
5	1	300,000

Table 5.33 Production capacity at engine plants

Auto assembly plant, a	Period, t	Production capacity, $PC3_{at}$
1	1	500,000
2	1	500,000
3	1	500,000
4	1	500,000
5	1	500,000
6	1	500,000
7	1	500,000
8	1	500,000
9	1	500,000
10	1	500,000
11	1	500,000
12	1	500,000
13	1	500,000

Table 5.34 Production capacity at auto assembly plants

Engine type, *n*	Product family, *q*	Auto assembly plant, *a*	Period, *t*	Average demand for engines, $AD6_{nqat}$ (units/day)	Average demand for vehicles, $AD7_{nqat}$ (units/day)	Storage space for engines $FC6_{nqat}$ (cu.ft./unit)	Storage space for vehicles $FC7_{nqat}$ (cu.ft./unit)
4	1	1	1	196	196	300,000	300,000
6	1	1	1	127	127	300,000	300,000
7	2	1	1	127	127	300,000	300,000
4	1	2	1	727	727	300,000	300,000
6	2	3	1	244	244	300,000	300,000
4	1	4	1	482	482	300,000	300,000
6	2	4	1	130	130	300,000	300,000
6	2	5	1	245	245	300,000	300,000
6	2	6	1	1,061	1,061	300,000	300,000
6	2	7	1	584	584	300,000	300,000
2	1	8	1	325	325	300,000	300,000
4	1	9	1	623	623	300,000	300,000
3	1	10	1	189	189	300,000	300,000
4	1	10	1	829	829	300,000	300,000
5	1	11	1	562	562	300,000	300,000
1	1	12	1	561	561	300,000	300,000
6	2	13	1	201	201	300,000	300,000

Table 5.35 Average demand for engines and vehicles at auto assembly plants

Remanufacturing center, *u*	Period, *t*	Average demand for rebuilt engines $AD8_{ut}$ (units/day)	Production capacity, $PC5_{ut}$ (units/day)	Storage space for rebuildable engines, $FC8_{ut}$ (cu.ft.)	Storage space for rebuilt engines, $FC9_{ut}$ (cu.ft.)
1	1	51	300,000	300,000	300,000
2	1	51	300,000	300,000	300,000

Table 5.36 Average demand for rebuilt engines, production capacity and storage space for rebuildable engines and rebuilt engines at remanufacturing centers

Vehicle with engine type, *n*	Product family, *q*	Dealership, *d*	Period, *t*	Average demand for vehicles, $AD9_{nqdt}$ (units/day)
1	1	1	1	561
2	1	1	1	325
3	1	1	1	189
4	1	1	1	2,857
5	1	1	1	562
6	1	1	1	127
6	2	1	1	2,465
7	2	1	1	127

Table 5.37 Average demand for vehicles at dealerships

Dealership, d	Period, t	Average demand for rebuilt engines, $AD10_{dt}$ (units/day)
1	1	102
1	2	102
1	3	102
1	4	102
1	5	102
1	6	102
1	7	102
1	8	102
1	9	102
1	10	102
1	11	102
1	12	102
1	13	102
1	14	102
1	15	102
1	16	102
1	17	102
1	18	102
1	19	102
1	20	102

Table 5.38 Average demand for rebuilt engines at dealerships

Collection center, c	Period, t	End-of-life vehicles collected, ELV_{ct} (units/day)[6]	Processing capacity, $PC4_{ct}$ (units/day)
1	1	2,988	300,000
2	1	2,988	300,000
3	1	3,983	300,000

Table 5.39 End-of-life vehicles collected and processing capacity at collection centers

5.2 Solution methodology

The proposed model is solved using Lingo 9.0 (Lingo System Inc., 2003). The Lingo program scripts have been provided in Appendix II. Given the size and the complexity of the problem, it is necessary to use a relaxed version of the model that

[6] Information is available at http://www.ford.com/en/company/about/sustainability/report/proData.htm#B http://www.systemdynamics.org

considers the large number of general integer variables as continuous variables. As mentioned in Winston (2004), the relaxed model gives a good approximation of the integer solution. The model has 9251 variables and 8104 constraints, and the typical solution time using the LINGO solver is between 5 to 11 seconds.

5.3 Detailed results

Table 5.40 shows the detailed results with respect to the operational costs of the chain over a period of a month. Figure 5.1 indicates the largest cost item is handling costs, the second item is labor costs and the next is purchasing cost.

Table 5.41 shows the amount of aluminum ingots purchased from suppliers, and the amount of recycled aluminum purchased from recycling centers, by the aluminum casting plant.

Table 5.42 shows the number of engines produced XM_{nqmt} and the average daily inventory of engines $XIV5_{nqmt}$ held at each engine plant. For example, the number of engine type n=1 in product family q=1 produced at engine plant m=1 is 11,220 per month and the average inventory of engines are 1,122 units per day.

Table 5.43 shows the number of vehicles produced $XUS3_{nqat}$ and the average daily inventory of vehicles $XIV7_{nqat}$ held at each auto assembly plant. For example, the number of vehicles with engine type n=4 in product family q=1 produced at auto assembly plant a=1 is 3,924 per month and the average inventory is 392 units per day.

Table 5.44 shows the number of flattened hulks sent from collection centers (decision points) to recycling centers $XPS4_{crkt}$, and the number of rebuildable engines sent from collection centers to remanufacturing centers $XPS6_{cukt}$.

The complete results are displayed in Appendix III.

TOTAL COST	***368,187,947***
CAPITAL COST(INVENTORY COSTS AND PURCHASING COSTS)	*20,757,297*
EXPENDITURE(LABOR, HANDLING, TRANSPORTATION, AND IN-TRANSIT COSTS)	*350,608,960*
Distribution of the Total Costs	
PURCHASING COSTS	
aluminum ingots purchased	2,423,017
recycled aluminum purchased	10,148,553
TOTAL PURCHASING COSTS	***12,571,570***
TRANSPORTATION COSTS	
from the aluminum casting plant to engine plants	96,956
from engine plants to auto assembly plants	1,914,420
from auto assembly plants to dealerships	7,809,845
from collection centers to recycling centers	903,440
from engine plants to remanufacturing centers	6,319
from collection centers to remanufacturing centers	26,283
from remanufacturing centers to dealerships	556,204
TOTAL TRANSPORTATION COSTS	***11,313,467***
INVENTORY COSTS	
aluminum ingots held at the aluminum casting plant	2,617
recycled aluminum held at the aluminum casting plant	10,960
engine parts held at the aluminum casting plant	90,249
engine parts held at engine plants	90,768
assembled engines held at engine plants	839,010
assembled engines held at auto assembly plants	854,654
vehicles held at auto assembly plants	6,292,573
rebuildable engines held at remanufacturing centers	0
rebuilt engines held at remanufacturing centers	4,896
TOTAL INVENTORY COSTS	***8,185,727***
IN-TRANSIT INVENTORY COSTS	
from the aluminum casting plant to engine plants	37,616
from engine plants to auto assembly plants	231,069
from auto assembly plants to dealerships	6,099,071
from collection centers to recycling centers	64,977
TOTAL IN-TRANSIT INVENTORY COSTS	***6,432,733***
LABOR COSTS	
regular time labor cost at the aluminum casting plant	127,745
overtime labor cost at the aluminum casting plant	0
regular time labor cost at engine plants	43,424,610
overtime labor cost at engine plants	3,796,215
hiring cost at engine plants	223,680
layoff cost at engine plants	0
regular time labor cost at remanufacturing centers	19,245,570
overtime labor cost at remanufacturing centers	0
TOTAL LABOR COSTS	***66,817,820***
HANDLING COSTS	
handling at auto assembly plants	204,159,900
handling at collection centers	61,885,040
TOTAL HANDLING COSTS	***266,044,940***
TOTAL REVENUE	***3,178,310***

Table 5.40 Cost components (dollars/month)

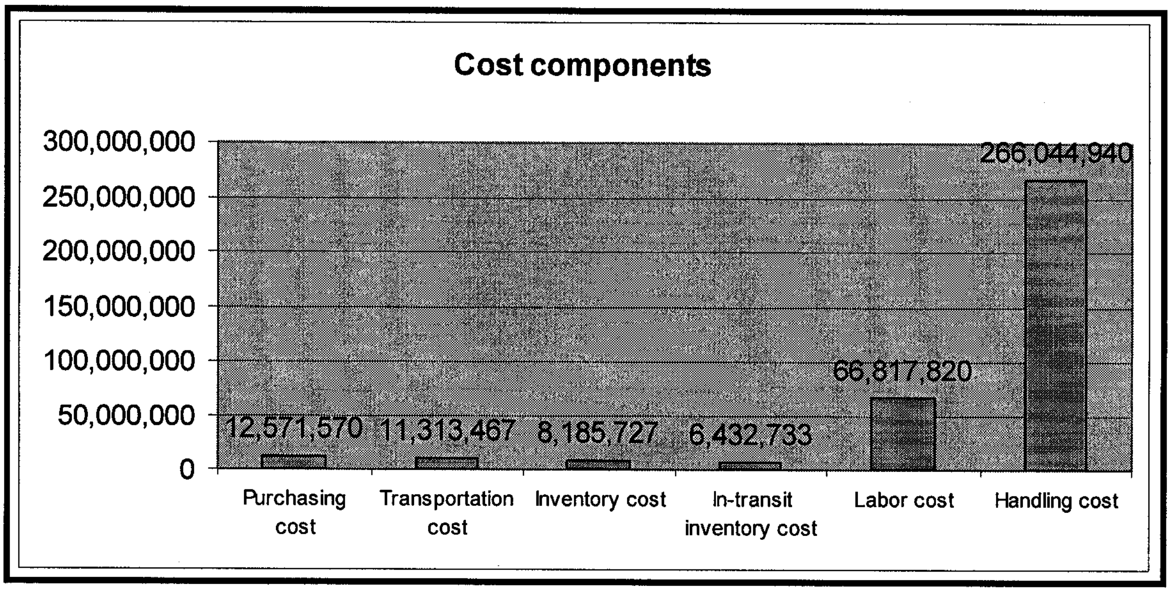

Figure 5.1 Cost components

Decision variables	Input	Amount, in lbs
$P1\sum_{s=1}^{S}\sum_{t=1}^{T} XC_{st}$	aluminum ingots	2,106,966
$P2\sum_{s=1}^{S}\sum_{t=1}^{T} XC_{st}$	recycled aluminum	11,939,474

Table 5.41 Amount of material purchased

Engine type, n	Product family, q	Engine plant, m	Production $\sum_{t=1}^{T} XM_{nqmt}$ (units/month)	Inventory $XIV5_{nqmt}$ (units/day)
1	1	1	11,220	1,122
2	1	1	6,500	650
3	1	2	3,780	378
4	1	2	42,600	4,260
4	1	3	14,540	1,454
5	1	4	11,240	1,124
6	1	5	2,540	254
6	2	5	49,300	4,930
7	2	5	2,540	254

Table 5.42 The number of engines produced and the average daily inventory of engines held at each engine plant

Engine type, n	Product family, q	Auto assembly plant, a	Production $\sum_{t=1}^{T} XUS3_{nqat}$ (units/month)	Inventory $XIV7_{nqat}$ (units/day)
4	1	1	3,924	392
6	1	1	2,540	254
7	2	1	2,540	254
4	1	2	15,267	1,454
6	2	3	4,392	488
4	1	4	10,122	964
6	2	4	2,730	260
6	2	5	5,145	490
6	2	6	21,220	2,122
6	2	7	11,994	1,168
2	1	8	6,500	650
4	1	9	11,251	1,246
3	1	10	3,780	378
4	1	10	16,580	1,658
5	1	11	11,240	1,124
1	1	12	11,220	1,122
6	2	13	3,819	402

Table 5.43 The number of vehicles with different kind of engines produced and the average inventory of vehicles held at each auto assembly plant

Number of flattened hulks $\sum_{c=1}^{C}\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{t=1}^{T} XPS4_{crkt}$ (units/month)	Number of rebuildable engines $\sum_{c=1}^{C}\sum_{u=1}^{U}\sum_{k=1}^{K}\sum_{t=1}^{T} XPS6_{cukt}$ (units/month)
199,180	35,800

Table 5.44 Products sent from decision points

CHAPTER 6 SENSITIVITY ANALYSIS

The sensitivity analysis of the model is divided into four sections: analysis with respect to weighting factors, levels of safety stock, lead times and demand.

6.1 Sensitivity analysis with respect to weighting factors

In this section, we examine the impact of cost components weighting factors on the model performance. The objective function is now converted to the flowing form by assigning a weighting factor to each cost component:

$$Objective\ function = WPU\sum_{s=1}^{S}\sum_{t=1}^{T}(P1\times CP1\times XC_{st} + P2\times CP2\times XC_{st})$$

$$+\ WT\ \{\sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR1_{smk}\ XTR1_{smkt} + \sum_{m=1}^{M}\sum_{a=1}^{A}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR2_{mak}\ XTR2_{makt}$$

$$+\sum_{a=1}^{A}\sum_{d=1}^{D}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR3_{adk}\ XTR3_{adkt} + \sum_{c=1}^{C}\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR4_{crk}\ XTR4_{crkt}$$

$$+\sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR5_{muk}\ XTR5_{mukt} + \sum_{c=1}^{C}\sum_{u=1}^{U}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR6_{cuk}\ XTR6_{cukt}$$

$$+\sum_{u=1}^{U}\sum_{d=1}^{D}\sum_{k=1}^{K}\sum_{t=1}^{T} CTR7_{udk}\ XTR7_{udkt}\ \}$$

$$+\ WI\{\sum_{s=1}^{S}\sum_{t=1}^{T} CP1\times IVRS_{s}\times STRS\times XIV1_{st}$$

$$+\sum_{s=1}^{S}\sum_{t=1}^{T} CP2\times IVRS_{s}\times STRRS\times XIV2_{st} + \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{s=1}^{S}\sum_{t=1}^{T} PP_{in} IVRS_{s} STPS\times XIV3_{inst}$$

$$+\sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{t=1}^{T} PP_{in} IVRM_{m} STPM\times XIV4_{inmt}$$

$$+\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{m=1}^{M}\sum_{t=1}^{T} PNE_{nq} IVRM_{m} STEM\times XIV5_{nqmt}$$

$$+\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{a=1}^{A}\sum_{t=1}^{T} PNE_{nq} IVRA_{a} STEA\times XIV6_{nqat} + \sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{a=1}^{A}\sum_{t=1}^{T} PV_{nq} IVRA_{a} STVA\times XIV7_{nqat}$$

$$+\sum_{u=1}^{U}\sum_{t=1}^{T} PNRE \times IVRU_u STNEU \times XIV8_{ut} + \sum_{u=1}^{U}\sum_{t=1}^{T} PRE \times IVRU_u STEU \times XIV9_{ut} \}$$

$$+ WIT \{ \sum_{s=1}^{S}\sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{t=1}^{T} PP_{in} IVRS_s LT1_{smk} XIT1_{sin\,mkt}$$

$$+\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{a=1}^{A}\sum_{k=1}^{K}\sum_{t=1}^{T} PNE_{nq} IVRM_m LT2_{mak} XIT2_{mnqakt}$$

$$+\sum_{a=1}^{A}\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{d=1d}^{D}\sum_{k=1}^{K}\sum_{t=1}^{T} PV_{nq} IVRA_a LT3_{adk} XIT3_{anqdkt}$$

$$+\sum_{c=1}^{C}\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{t=1}^{T} PFH \times IVRC_c LT4_{crk} XIT4_{crkt} \}$$

$$+ WL \{ \sum_{s=1}^{S}\sum_{t=1}^{T} CRLS_{st} XRLS_{st} + \sum_{s=1}^{S}\sum_{t=1}^{T} COLS_{st} XOLS_{st} + \sum_{m=1}^{M}\sum_{t=1}^{T} CRLM_{mt} XRLM_{mt}$$

$$+\sum_{m=1}^{M}\sum_{t=1}^{T} COLM_{mt} XOLM_{mt} + \sum_{m=1}^{M}\sum_{t=1}^{T} CHLM_{mt} XHLM_{mt} + \sum_{m=1}^{M}\sum_{t=1}^{T} CLLM_{mt} XLLM_{mt}$$

$$+\sum_{u=1}^{U}\sum_{t=1}^{T} CRLU_{ut} XRLU_{ut} + \sum_{u=1}^{U}\sum_{t=1}^{T} COLU_{ut} XOLU_{ut} \}$$

$$+ WH \{ \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{a=1}^{A}\sum_{k=1}^{K}\sum_{t=1}^{T} CH1_{nqa} XPS2_{mnqakt} + \sum_{c=1}^{C}\sum_{t=1}^{T} (ELV_{ct} + WRT) \}$$

$$-\sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{u=1}^{U}\sum_{k=1}^{K}\sum_{t=1}^{T} CP3_{miuk} XPS5_{miukt}$$

WPU: weighting factor for purchasing cost

WT: weighting factor for transportation cost

WI: weighting factor for inventory cost

WIT: weighting factor for in-transit inventory cost

WL: weighting factor for labor cost

WH: weighting factor for handling cost

This implies that the various elements of the objective function are now assigned weights that reflect the relative importance of each element, as opposed to the original objective function where all the elements are equally important.

We examined three scenarios. We started with the same weighting factors for the transportation cost and the inventory cost — a configuration that has been gaining more interest in the industry. In the next scenario we changed the weighting factor for the transportation cost so that it was twice that of the inventory cost. Finally, in the last scenario we reversed the above scheme and made the weighting factor for the inventory cost twice that of the transportation cost. We assigned the same weighting factors to other cost elements.

Table 6.1 shows the effect of the changes in the weighting factor on the cost elements.

When we put a higher weight on the transportation costs, and a lower weight on the inventory costs the total transportation cost decreased, while the total inventory cost went up — unlike the results we got when both costs shared the same weighting factors. If more importance was placed on the transportation costs, the system chose a lower transportation cost with a higher lead time, which prompted the in-transit inventory cost to increase.

When a lower weighting factor was placed on the transportation costs and a higher weighting factor on the inventory costs, the results revealed a higher transportation cost and a lower inventory cost — unlike the results when both costs shared the same weighting factors. If less importance was placed on the transportation cost, the system chose faster transportation with a shorter lead time, which prompted the in-transit inventory cost to decrease as well.

6.2 Sensitivity analysis with respect to levels of safety stock

Table 6.2 shows the effect of changes in the levels of safety stock on the costs. At present, Ford keeps a safety stock equal to a two-day production level. In this section we changed the safety stock level at the aluminum casting plant, engine plants and auto assembly plants at the same time.

Figure 6.1 illustrates how changing the levels of safety stock affects the total costs. As can be saw, when the level of safety stock decreases from 2 days to 1 day, the total costs decreases from about \$ 368 million to about \$ 362 million (a decrease of about 1.67%), due, mainly, to the lower levels of inventories at the aluminum casting plant, the

WEIGHTING FACTORS	Current solution	Scenario 1	Scenario 2	Scenario 3
for transportation costs	–	30%	40%	20%
for inventory costs	–	30%	20%	40%
for purchasing costs	–	10%	10%	10%
for in-transit inventory costs	–	10%	10%	10%
for labor costs	–	10%	10%	10%
for handling costs	–	10%	10%	10%
TRANSPORTATION COSTS				
from the aluminum casting plant to engine plants	96,956	96,956	96,956	96,956
from engine plants to auto assembly plants	1,914,420	1,420,512	1,358,627	1,557,649
from auto assembly plants to dealerships	7,809,845	4,536,852	3,764,811	4,934,421
from collection centers to recycling centers	903,440	891,021	891,021	891,021
from engine plants to remanufacturing centers	6,319	6,319	6,319	6,319
from collection centers to remanufacturing centers	26,283	26,283	26,283	26,283
from remanufacturing centers to dealerships	556,204	556,204	556,078	556,078
TOTAL TRANSPORTATION COSTS	***11,313,467***	***7,534,147***	***6,700,095***	***8,081,146***
IN-TRANSIT INVENTORY COSTS				
from the aluminum casting plant to engine plants	37,616	37,616	37,616	37,616
from engine plants to auto assembly plants	231,069	1,975,308	1,739,406	1,133,657
from auto assembly plants to dealerships	6,099,071	7,378,120	7,993,050	7,902,057
from collection centers to recycling centers	64,977	90,525	90,525	64,977
TOTAL IN-TRANSIT INVENTORY COSTS	***6,432,733***	***9,481,569***	***9,860,597***	***9,138,307***
INVENTORY COSTS				
aluminum ingots held at the aluminum casting plant	2,617	2,617	2,617	2,617
recycled aluminum held at the aluminum casting plant	10,960	10,960	10,960	10,960
engine parts held at the aluminum casting plant	90,249	90,249	90,249	90,249
engine parts held at engine plants	90,768	90,768	90,768	90,768
assembled engines held at engine plants	839,010	839,010	839,010	839,010
assembled engines held at auto assembly plants	854,654	1,502,484	2,490,256	1,238,869
vehicles held at auto assembly plants	6,292,573	6,319,633	6,319,633	6,292,573
rebuildable engines held at remanufacturing centers	0	0	0	0
engines rebuilt held at remanufacturing centers	4,896	4,896	4,896	4,896
TOTAL INVENTORY COSTS	***8,185,727***	***8,860,617***	***9,848,389***	***8,569,942***
ALL OTHER COSTS	***345,434,330***	***345,434,330***	***345,434,330***	***345,434,330***
TOTAL REVENUE (dollars/month)	***3,178,310***	***3,178,310***	***3,178,310***	***3,178,310***
TOTAL COST(dollars/month)	***368,187,947***	***368,132,353***	***368,665,101***	***368,045,415***
CAPITAL COST(INVENTORY COSTS AND PURCHASING COSTS)	*20,757,297*	*21,432,187*	*22,419,959*	*21,141,512*
EXPENDITURE(LABOR, HANDLING, TRANSPORTATION, AND IN-TRANSIT COSTS)	*350,608,960*	*349,878,476*	*349,423,452*	*350,082,213*

Table 6.1 Effect of changes in weighting factors on various cost elements

engine plants and the auto assembly plants.

LEVELS OF SAFETY STOCK	2 DAYS	1.5 DAYS	1 DAY
TRANSPORTATION COSTS			
from the aluminum casting plant to engine plants	96,956	96,956	96,956
from engine plants to auto assembly plants	1,914,420	1,914,420	1,926,872
from auto assembly plants to dealerships	7,809,845	7,809,845	7,809,845
from collection centers to recycling centers	903,440	903,440	903,440
from engine plants to remanufacturing centers	6,319	6,319	6,319
from collection centers to remanufacturing centers	26,283	26,283	26,283
from remanufacturing centers to dealerships	556,204	556,204	556,204
TOTAL TRANSPORTATION COSTS	***11,313,467***	***11,313,467***	***11,325,919***
INVENTORY COSTS			
aluminum ingots held at the aluminum casting plant	2,617	1,472	654
recycled aluminum held at the aluminum casting plant	10,960	6,165	2,740
engine parts held at the aluminum casting plant	90,249	50,765	22,562
engine parts held at engine plants	90,768	51,154	22,822
assembled engines held at engine plants	839,010	471,943	209,752
assembled engines held at auto assembly plants	854,654	483,676	224,168
vehicles held at auto assembly plants	6,292,573	3,539,572	1,573,143
rebuildable engines held at remanufacturing centers	0	0	0
engines rebuilt held at remanufacturing centers	4,896	4,896	4,896
TOTAL INVENTORY COSTS	***8,185,727***	***4,609,643***	***2,060,737***
IN-TRANSIT INVENTORY COSTS			
from the aluminum casting plant to engine plants	37,616	37,616	37,616
from engine plants to auto assembly plants	231,069	231,069	211,287
from auto assembly plants to dealerships	6,099,071	6,099,071	6,099,071
from collection centers to recycling centers	64,977	64,977	64,977
TOTAL IN-TRANSIT INVENTORY COSTS	***6,432,733***	***6,432,733***	***6,412,951***
ALL OTHER COSTS (dollars/month)	***345,434,330***	***345,434,330***	***345,434,330***
TOTAL REVENUE (dollars/month)	***3,178,310***	***3,178,310***	***3,178,310***
TOTAL COST (dollars/month)	***368,187,947***	***364,611,863***	***362,055,627***
CAPITAL COST(INVENTORY COSTS AND PURCHASING COSTS)	*20,757,297*	*17,181,213*	*14,632,307*
EXPENDITURE(LABOR, HANDLING, TRANSPORTATION, AND IN-TRANSIT COSTS)	*350,608,960*	*350,608,960*	*350,601,630*

Table 6.2 Effect of changes in levels of safety stock

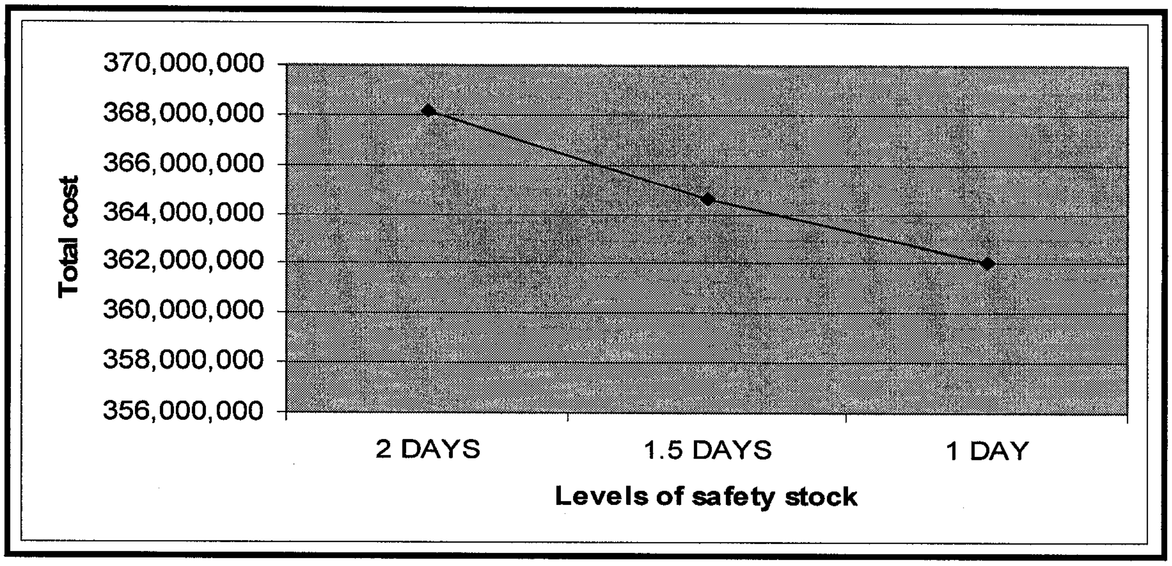

Figure 6.1 Effect of changes in levels of safety stock on total cost

6.3 Sensitivity analysis with respect to lead times

6.3.1 Sensitivity analysis with respect to lead times between the aluminum casting plant and engine plants

Table 6.3 shows the different lead times from the aluminum casting plant to engine plants corresponding to the current conditions, scenario 1, and scenario 2. For example, the current lead time from the aluminum casting plant s=1 to engine plant m=2, using transport mode k=1, is 3 days; under scenario 1 it is 2 days and under scenario 2 it is 1 day.

Aluminum casting plant, s	Engine plant, m	Transport Mode, k	Current (days)	Scenario 1 (days)	Scenario 2 (days)
1	2	1	3	2	1
1	2	2	1	1	1
1	3	1	5	3	1
1	3	2	2	1	1
1	5	1	5	3	1
1	5	2	1	1	1

Table 6.3 Lead times from aluminum casting plant to engine plants corresponding to current conditions, scenario 1, and scenario 2

Table 6.4 shows how changes in the transportation lead times between the aluminum casting plant and the engine plants affect the total cost relative to the current solution. Under scenario1 the total costs decrease by $3,504,532 per month (or about 1%). The decrease is the result of lower inventory cost as well as lower in-transit inventory costs. Under scenario 2, however, there is no further reduction in the total costs, i.e., further decreases in the transportation lead times have no effect on the total costs, as shown in Figure 6.2.

LEAD TIMES BETWEEN THE ALUMINUM CASTING PLANT AND ENGINE PLANTS	Current solution	Scenario 1	Scenario 2
TRANSPORTATION COST			
from the aluminum casting plant to engine plants	96,956	96,956	96,956
from engine plants to auto assembly plants	1,914,420	1,634 ,359	1,634 ,359
from auto assembly plants to dealerships	7,809,845	4,600,3947	4,600,3947
from collection centers to recycling centers	903,440	903,440	903,440
from engine plants to remanufacturing centers	6,319	6,319	6,319
from collection centers to remanufacturing centers	26,283	26,283	26,283
from remanufacturing centers to dealerships	556,204	556,204	556,204
TOTAL TRANSPORTATION COSTS	***11,313,427***	***7,824,468***	***7,824,468***
INVENTORY COSTS			
aluminum ingots held at the aluminum casting plant	2,617	2,617	2,617
recycled aluminum held at the aluminum casting plant	10,960	10,960	10,960
engine parts held at the aluminum casting plant	90,249	90,249	90,249
engine parts held at engine plants	90,768	90,249	90,249
assembled engines held at engine plants	839,010	839,010	839,010
assembled engines held at auto assembly plants	854,654	854,116	854,654
vehicles held at auto assembly plants	6,292,573	6,292,573	6,292,573
rebuildable engines held at remanufacturing centers	0	0	0
engines rebuilt held at remanufacturing centers	4,896	4,896	4,896
TOTAL INVENTORY COSTS	***8,185,727***	***8,184,670***	***8,185,208***
IN-TRANSIT INVENTORY COSTS			
from the aluminum casting plant to engine plants	37,616	22,562	22,562
from engine plants to auto assembly plants	231,069	231,607	231,069
from auto assembly plants to dealerships	6,099,071	6,099,071	6,099,071
from collection centers to recycling centers	64,977	64,977	64,977
TOTAL IN-TRANSIT INVENTORY COSTS	***6,432,733***	***6,418,217***	***6,417,679***
ALL OTHER COSTS	***345,434,330***	***345,434,330***	***345,434,330***
TOTAL REVENUE	***3,178,310***	***3,178,310***	***3,178,310***
TOTAL COST	***368,187,907***	***364,683,375***	***364,683,375***
CAPITAL COST (INVENTORY COSTS AND PURCHASING COST)	*20,757,297*	*20,756,240*	*20,756,778*
EXPENDITURE(LABOR,HANDLING,TRANSPORTATION, IN-TRANSIT COST)	*350,608,920*	*347,105,445*	*347,104,907*

Table 6.4 Effect of changes in lead times between the aluminum casting plant and engine plants (dollars/month)

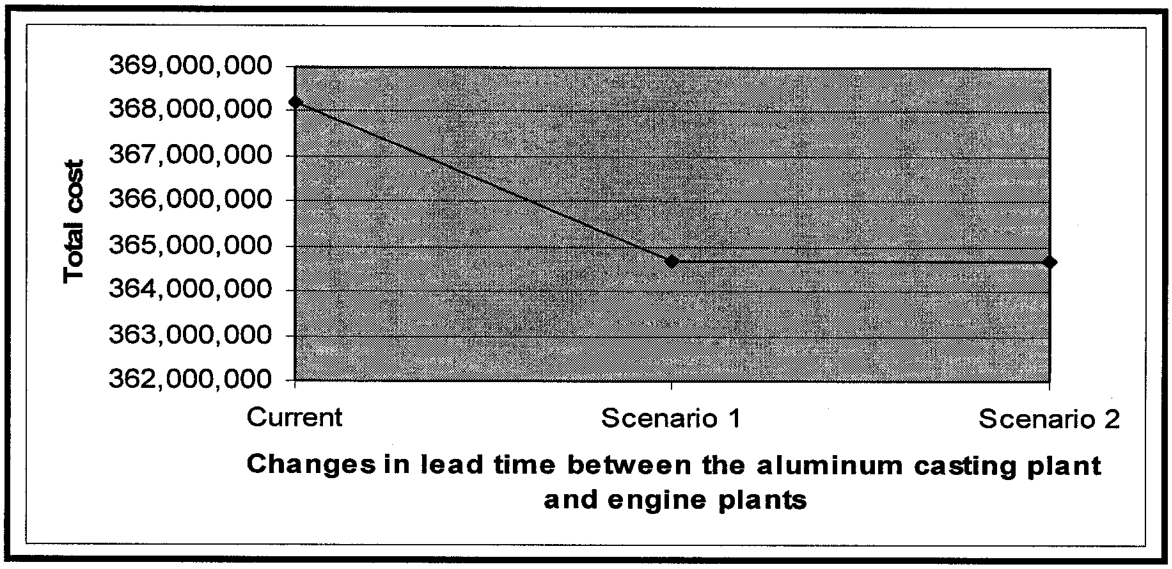

Figure 6.2 Effect of changes in lead times between the aluminum casting plant and engine plants on total cost

6.3.2 Sensitivity analysis with respect to lead times between engine plants and auto assembly plants

Table 6.5 shows the different lead times from engine plants to auto assembly plants under the current conditions, and under scenarios 1 and 2. For example, the current lead time from engine plant m=1 to auto assembly plant a=8, using transport mode k=1, is 5 days; under scenario 1 it is 3 days, and under scenario 2 it is 1 day.

Table 6.6 and Figure 6.3 show how changes in the transportation lead times between engine plants and auto assembly plants affect the total cost relative to the current solution. When the lead times are reduced according to scenario 1, the total costs decrease by $135,128 (or about 0.04%); when scenario 2 is in force, the total costs decrease by $479,352 (or about 0.13%). The effect is not significant, which implies that the model is not very sensitive to lead time variations.

6.3.3 Sensitivity analysis with respect to lead times between auto assembly plants and dealerships

Table 6.7 shows the different lead times from auto assembly plants to dealerships under the current condition, and under scenarios 1 and 2. For example, the current lead time from auto assembly plant a=1 to dealership d=1, using transport mode k=1, is 3 days, under scenario 1 it is 2 days, and under scenario 2 it is 1 day.

Table 6.8 and Figure 6.4 show how changes in the transportation lead times between auto assembly plants and dealerships affect the total cost relative to the current solution. Changes in transportation lead times reduce the total cost when they are changed according to scenario 1. The total cost reduction is $3,747,661, or about 1%. When the lead times are changed according to scenario 2, the total costs decrease by $6,695,791, or about 1.8%. Once again, the effect is not significant, and is an indication of the model's robustness.

Engine plant, m	Auto assembly plant, a	Transport mode, k	Current (days)	Scenario 1 (days)	Scenario 2 (days)
1	8	1	5	3	1
1	8	2	0	0	0
1	12	1	1	1	1
1	12	2	0	0	0
2	1	1	5	3	1
2	1	2	1	1	1
2	4	1	3	2	1
2	4	2	0	0	0
2	9	1	8	6	4
2	9	2	0	0	0
2	10	1	8	6	4
2	10	2	0	0	0
3	2	1	1	1	1
3	2	2	0	0	0
4	11	1	1	1	1
4	11	2	0	0	0
5	1	1	1	1	1
5	1	2	0	0	0
5	3	1	1	1	1
5	3	2	0	0	0
5	4	1	2	1	1
5	4	2	0	0	0
5	5	1	5	3	1
5	5	2	0	0	0
5	6	1	3	2	1
5	6	2	1	1	1
5	7	1	2	1	1
5	7	2	0	0	0
5	13	1	1	1	1
5	13	2	0	0	0

Table 6.5 Lead times from engine plants to auto assembly plants corresponding to current conditions, scenario 1, and scenario 2

LEAD TIMES BETWEEN ENGINE PLANTS AND AUTO ASSEMBLY PLANTS	Current solution	Scenario 1	Scenario 2
TRANSPORTATION COST			
from the aluminum casting plant to engine plants	96,956	96,956	96,956
from engine plants to auto assembly plants	1,914,420	1,796 ,259	1,493,726
from auto assembly plants to dealerships	7,809,845	7,820,398	7,820,398
from collection centers to recycling centers	903,440	903,440	903,440
from engine plants to remanufacturing centers	6,319	6,319	6,319
from collection centers to remanufacturing centers	26,283	26,283	26,283
from remanufacturing centers to dealerships	556,204	556,204	556,204
TOTAL TRANSPORTATION COSTS	***11,313,467***	***11,205,859***	***10,903,326***
INVENTORY COST			
aluminum ingots held at the aluminum casting plant	2,617	2,617	2,617
recycled aluminum held at the aluminum casting plant	10,960	10,960	10,960
engine parts held at the aluminum casting plant	90,249	90,249	90,249
engine parts held at engine plants	90,768	90,768	90,768
assembled engines held at engine plants	839,010	839,010	839,010
assembled engines held at auto assembly plants	854,654	859,826	842,918
vehicles held at auto assembly plants	6,292,573	6,292,573	6,292,573
rebuildable engines held at remanufacturing centers	0	0	0
engines rebuilt held at remanufacturing centers	4,896	4,896	4,896
TOTAL INVENTORY COSTS	***8,185,727***	***8,190,899***	***8,173,991***
IN-TRANSIT INVENTORY COST			
from the aluminum casting plant to engine plants	37,616	37,616	37,616
from engine plants to auto assembly plants	231,069	227,568	207,607
from auto assembly plants to dealerships	6,099,071	6,069,880	6,065,058
from collection centers to recycling centers	64,977	64,977	64,977
TOTAL IN-TRANSIT INVENTORY COSTS	***6,432,733***	***6,400,041***	***6,375,258***
ALL OTHER COSTS	***345,434,330***	***345,434,330***	***345,434,330***
TOTAL REVENUE	***3,178,310***	***3,178,310***	***3,178,310***
TOTAL COST	***368,187,947***	***368,052,819***	***367,708,595***
CAPITAL COST (INVENTORY COSTS AND PURCHASING COSTS)	*20,757,297*	*20,762,469*	*20,745,561*
EXPENDITURE(LABOR, HANDLING, TRANSPORTATION, AND IN-TRANSIT COSTS)	*350,608,960*	*350,468,660*	*350,141,344*

Table 6.6 Effect of changes in lead times between engine plants and auto assembly plants (dollars/month)

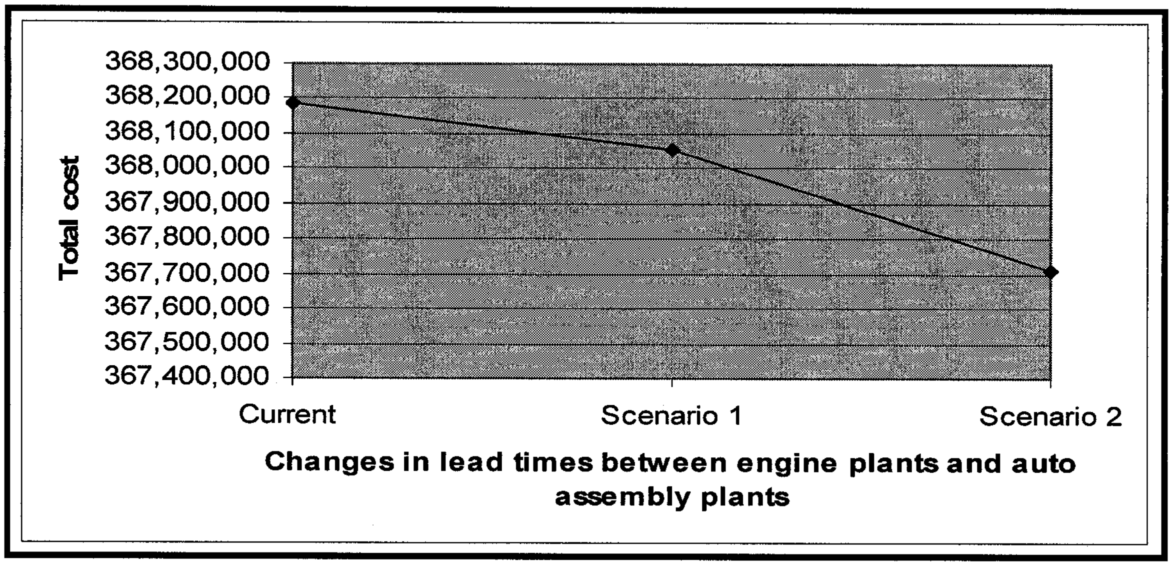

Figure 6.3 Effect of changes in lead times between engine plants and auto assembly plants on total cost

Auto assembly plant, *a*	Dealership, *d*	Mode, *k*	Current (days)	Scenario 1 (days)	Scenario 2 (days)
1	1	1	3	2	1
1	1	2	1	1	1
2	1	1	2	1	1
2	1	2	1	1	1
3	1	1	10	8	6
3	1	2	2	1	1
4	1	1	1	1	1
4	1	2	1	1	1
5	1	1	1	1	1
5	1	2	1	1	1
6	1	1	0	0	0
6	1	2	1	1	1
7	1	1	5	3	1
7	1	2	2	1	1
8	1	1	1	1	1
8	1	2	0	0	0
9	1	1	3	2	1
9	1	2	1	1	1
10	1	1	4	3	2
10	1	2	0	0	0
11	1	1	0	0	0
11	1	2	0	0	0
12	1	1	0	0	0
12	1	2	0	0	0
13	1	1	3	2	1
13	1	2	1	1	1

Table 6.7 Lead times from auto assembly plants to dealerships corresponding to current conditions, scenario 1, and scenario 2

LEAD TIMES BETWEEN AUTO ASSEMBLY PLANTS AND DEALERSHIPS	Current solution	Scenario 1	Scenario 2
TRANSPORTATION COSTS			
from the aluminum casting plant to engine plants	96,956	96,956	96,956
from engine plants to auto assembly plants	1,914,420	1,905,738	1,926,586
from auto assembly plants to dealerships	7,809,845	5,745,580	5,682,330
from collection centers to recycling centers	903,440	903,440	903,440
from engine plants to remanufacturing centers	6,319	6,319	6,319
from collection centers to remanufacturing centers	26,283	26,283	26,283
from remanufacturing centers to dealerships	556,204	556,204	556,204
TOTAL TRANSPORTATION COSTS	***11,313,467***	***9,240,520***	***9,198,118***
INVENTORY COSTS			
aluminum ingots held at the aluminum casting plant	2,617	2,617	2,617
recycled aluminum held at the aluminum casting plant	10,960	10,960	10,960
engine parts held at the aluminum casting plant	90,249	90,249	90,249
engine parts held at engine plants	90,768	90,768	90,768
assembled engines held at engine plants	839,010	839,010	839,010
assembled engines held at auto assembly plants	854,654	853,623	856,110
vehicles held at auto assembly plants	6,292,573	6,292,573	6,292,573
rebuildable engines held at remanufacturing centers	0	0	0
engines rebuilt held at remanufacturing centers	4,896	4,896	4,896
TOTAL INVENTORY COSTS	***8,185,727***	***8,184,696***	***8,187,183***
IN-TRANSIT INVENTORY COSTS			
from the aluminum casting plant to engine plants	37,616	37,616	37,616
from engine plants to auto assembly plants	231,069	231,820	197,852
from auto assembly plants to dealerships	6,099,071	4,424,637	1,550,390
from collection centers to recycling centers	64,977	64,977	64,977
TOTAL IN-TRANSIT INVENTORY COSTS	***6,432,733***	***4,759,050***	***1,850,835***
ALL OTHER COSTS	***345,434,330***	***345,434,330***	***345,434,330***
TOTAL REVENUE	***3,178,310***	***3,178,310***	***3,178,310***
TOTAL COST	***368,187,947***	***364,440,286***	***361,492,156***
CAPITAL COST (INVENTORY COSTS AND PURCHASING COSTS)	*20,757,297*	*20,756,266*	*20,758,753*
EXPENDITURE(LABOR,HANDLING, TRANSPORTATION, AND IN-TRANSIT COSTS)	*350,608,960*	*346,862,330*	*343,911,713*

Table 6.8 Effect of changes in lead times between auto assembly plants and dealerships (dollars/month)

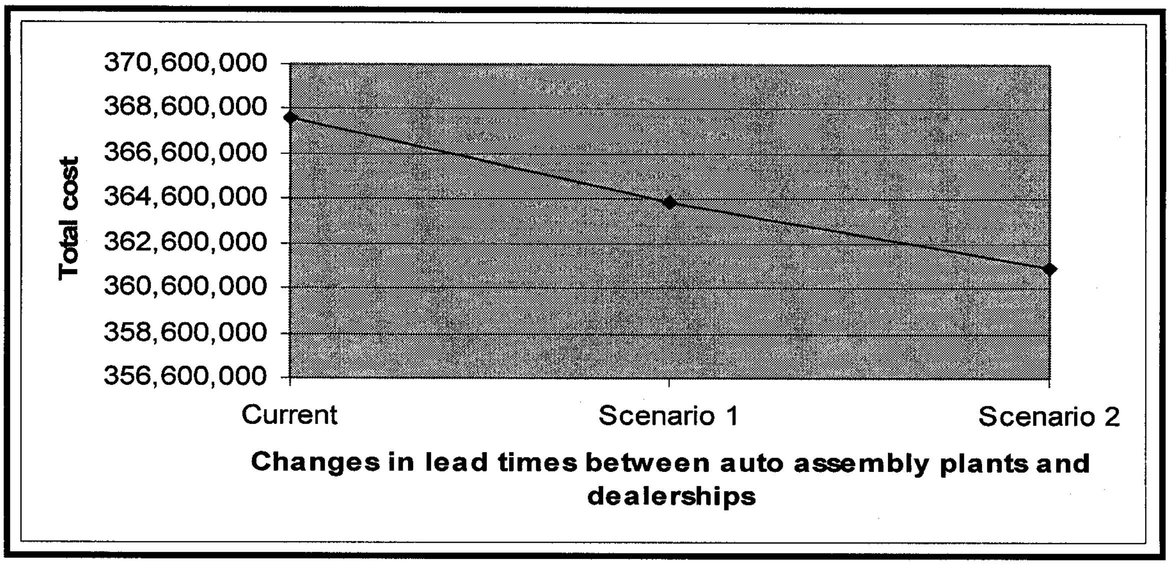

Figure 6.4 Effect of changes in lead times between auto assembly plants and dealerships on total cost

6.4 Sensitivity analysis with respect to demand for vehicles

Table 6.9 and Figure 6.5 show how changes in the demand for vehicles affect the total cost relative to the current solution. When the demand is reduced by 5% and 10%, the corresponding total cost reductions are $14,467,503 and $29,071,698, or about 3.9% and 7.9% respectively. When the demand is increased by 5% and 10%, the total costs increase by $14,556,716 and $29,364,194 respectively, or about 4.0% and 8.0%. The changes in the total cost come from the changes in purchasing costs, transportation costs, inventory costs, in-transit inventory costs, labor costs and handling costs.

6.5 Summary of sensitivity analysis experiments

It has been noted that the model is not very sensitive to changes that occur within the selected parameters. In most cases, decreases in the total cost remain well below 2%. Therefore, the model displays a significant degree of robustness. The sensitivity analyses of the proposed model with respect to the selected parameters may be summarized as follows:

1. Industry decision makers would like to put more emphasis on minimizing expenditures on the transportation costs compared to inventory costs, for example. With this in mind, we introduced the weighting factors in a way that

DEMAND	Current	-10%	-5%	+5%	+10%
PURCHASING COSTS (dollars/month)					
aluminum ingots purchased	2,423,013	2,180,169	2,303,103	2,542,916	2,665,853
recycled aluminum purchased	10,148,557	9,131,432	9,646,329	10,650,763	11,165,674
TOTAL PURCHASING COSTS	*12,571,570*	*11,311,601*	*11,949,432*	*13,193,679*	*13,831,527*
TRANSPORTATION COSTS (dollars/month)					
from the aluminum casting plant to engine plants	96,956	87,239	92,159	101,754	106,674
from engine plants to auto assembly plants	1,914,420	1,723,584	1,819,522	2,009,706	2,105,945
from auto assembly plants to dealerships	7,809,845	7,033,085	7,423,040	8,197,840	8,588,295
from collection centers to recycling centers	903,440	903,440	903,440	903,440	903,440
from engine plants to remanufacturing centers	6,319	6,319	6,319	6,319	6,319
from collection centers to remanufacturing centers	26,283	26,283	26,283	26,283	26,283
from remanufacturing centers to dealerships	556,204	556,204	556,204	556,204	556,204
TOTAL TRANSPORTATION COSTS	*11,313,467*	*10,336,154*	*10,826,967*	*11,801,546*	*12,293,160*
INVENTORY COSTS (dollars/month)					
aluminum ingots held at the aluminum casting plant	2,617	2,355	2,487	2,746	2,879
recycled aluminum held at the aluminum casting plant	10,960	9,862	10,418	11,503	12,059
engine parts held at the aluminum casting plant	90,249	81,204	85,783	94,715	99,294
engine parts held at engine plants	90,768	81,671	86,276	95,260	99,865
assembled engines held at engine plants	839,010	755,301	797,507	880,677	922,946
assembled engines held at auto assembly plants	854,654	769,384	811,863	896,532	939,554
vehicles held at auto assembly plants	6,292,573	5,664,754	5,981,300	6,605,078	6,922,092
rebuildable engines held at remanufacturing centers	0	0	0	0	0
engines rebuilt held at remanufacturing centers	4,896	4,896	4,896	4,896	4,896
TOTAL INVENTORY COSTS	*8,185,727*	*7,369,427*	*7,780,530*	*8,591,407*	*9,003,585*
IN-TRANSIT INVENTORY COSTS (dollars/month)					
from the aluminum casting plant to engine plants	37,616	33,843	35,754	39,478	41,388
from engine plants to auto assembly plants	231,069	208,056	220,157	243,139	254,790
from auto assembly plants to dealerships	6,099,071	5,488,379	5,798,803	6,399,601	6,710,141
from collection centers to recycling centers	64,977	64,977	64,977	64,977	64,977
TOTAL IN-TRANSIT INVENTORY COSTS	*6,432,733*	*5,795,255*	*6,119,691*	*6,747,195*	*7,071,296*
LABOR COSTS (dollars/month)					
regular time labor cost at the aluminum casting plant	127,745	114,942	121,424	134,066	140,548
overtime labor cost at the aluminum casting plant	0	0	0	0	0
regular time labor cost at engine plants	43,424,610	41,131,670	42,726,700	44,072,100	44,732,070
overtime labor cost at engine plants	3,796,215	1,322,400	2,185,500	4,829,490	5,880,735
hiring labor cost at engine plants	223,680	0	0	1,130,880	2,051,520
layoff labor cost at engine plants	0	0	0	0	0
regular time labor cost at remanufacturing centers	19,245,570	19,245,570	19,245,570	19,245,570	19,245,570
overtime labor cost at remanufacturing centers	0	0	0	0	0
TOTAL LABOR COSTS	*66,817,820*	*61,814,582*	*64,279,194*	*69,412,106*	*72,050,443*
HANDLING COSTS (dollars/month)					
handling at auto assembly plants	204,159,900	183,782,500	194,057,900	214,292,000	224,595,400
handling at collection centers	61,885,040	61,885,040	61,885,040	61,885,040	61,885,041
TOTAL HANDLING COSTS	*266,044,940*	*245,667,540*	*255,942,940*	*276,177,040*	*286,480,441*
TOTAL REVENUE	*3,178,310*	*3,178,310*	*3,178,310*	*3,178,310*	*3,178,311*
TOTAL COST(dollars/month)	368,187,947	339,116,249	353,720,444	382,744,663	397,552,141
CAPITAL COST(INVENTORY COSTS AND PURCHASING COST)	*20,757,297*	*18,681,028*	*19,729,962*	*21,785,086*	*22,835,112*
EXPENDITURE(LABOR, HANDLING, TRANSPORTATION, IN-TRANSIT COST)	*350,608,960*	*323,613,531*	*337,168,792*	*364,137,887*	*377,895,340*

Table 6.9 Effect of changes in demand for vehicles

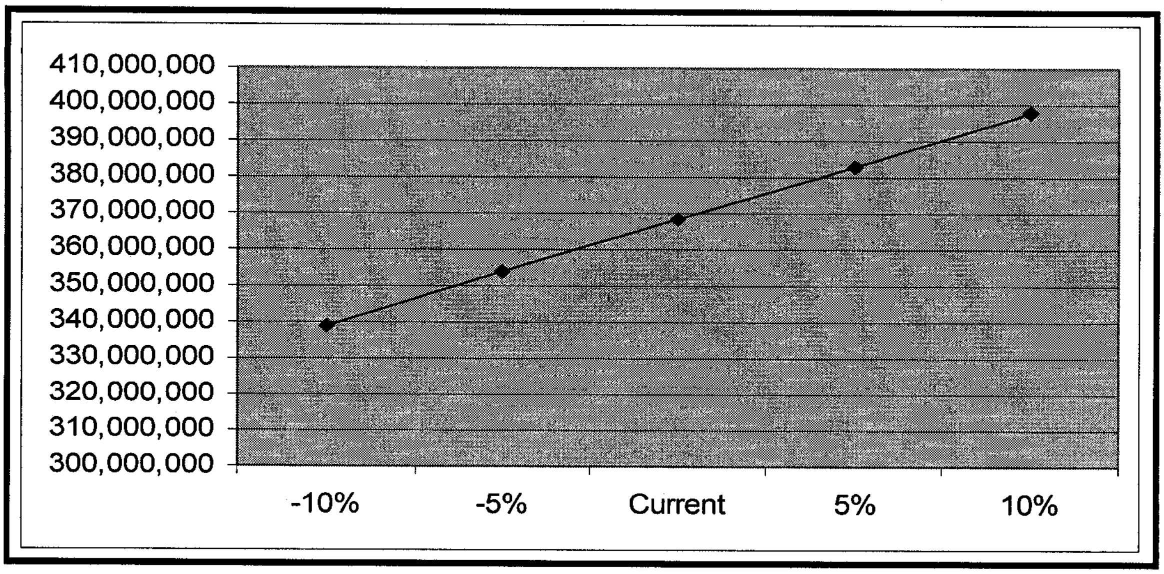

Figure 6.5 Effect of changes in demand for vehicles

allows for achieving this goal.

2. We examined the effects of the lower levels of safety stock kept at the aluminum casting plant, the engine plants and the auto assembly plants. Decision makers can save $ 6,132,320 per month by keeping a one-day production level instead of the usual two-day production level. Keeping less inventory while still meeting customer demands will allow decision makers to decrease the total costs by about 1.67%.
3. Changes in transportation lead times between the aluminum casting plant and the engine plants, between the engine plants and the auto assembly plants, and between the auto assembly plants and dealerships cause the total costs to decrease slightly in some scenarios and to remain unchanged in others. As mentioned already, the changes are fairly insignificant, and they indicate that the model, as a decision making tool, is a robust one.
4. Decreases in demand for vehicles cause the total cost to decrease and vice versa. The rate of change in the total cost is almost 1% for a 1% change in the demand. The changes in the total cost as a result of the demand changes are greater than those of other factors.

CHAPTER 7 SUMMARY AND FUTURE DIRECTIONS

This chapter summarizes the major contributions and conclusions of this thesis, then presents suggestions for the direction of future research.

7.1 Summary

Supply chain management is drawing great interest, both commercially and academically. Legal obligations and profit incentives aimed at recovering value from returned products have established a need for efficient supply chain designs. Establishing a manufacturer-wide, closed-loop supply chain would support the treatment of future end-of-life vehicles. While many studies have focused on reverse logistics, active research of the planning and optimization of reverse logistics systems for network design has been limited.

The proposed model is a multi-stage, multi-period, multi-product closed-loop supply chain that includes purchasing, production, and end-of-life products recycling and remanufacturing. The proposed general integer linear programming model can be used for any auto manufacturers, and is easily adapted to real-life scenarios by adding or removing any relevant constraints. It provides the users with a valuable and effective business decision making tool and was verified by the operations of Ford Motor Company with some real data extracted from industry sources.

7.2 Conclusions

From a purely computational point of view, the proposed model solves for 9251 variables and 8104 constraints. The typical solution time using the LINGO solver is between 5 to 11 seconds.

A sensitivity analysis was performed to examine the impact of the changes in the cost components weighting factors, the levels of safety stock, the lead times and the demand on the total costs. By assigning different weights to the transportation cost and the inventory cost, the relative importance of these two cost components may be assessed.

Placing more weight on the transportation cost than on the inventory cost causes the total cost to increase slightly relative to the original results, while placing more weight on the inventory cost than on the transportation cost caused the total cost to decrease slightly relative to the original results.

Decreasing the levels of safety stock at the aluminum casting plant, the engine plants, and the auto assembly plants resulted in the reduction of the total cost by about 6,132,320 per month, or approximately 1.67%.

Changes in the transportation lead times -- between the aluminum casting plant and the engine plants, the engine plants and the auto assembly plants, and the auto assembly plants and the dealerships—cause the total costs to decrease slightly in a non-linear manner. The decrease is well below 1%.

Decreases in the demand cause the total costs to decrease and vice versa. The changes in the total cost as a result of the demand changes are greater than those of other factors considered in this research.

The main contribution of the thesis is the development of a model for the planning, optimization and integration of the forward and reverse distribution networks in the context of an aluminum engine manufacturing and recycling. The model encompasses the engine manufacturing process including the purchasing, production, and end-of-life product recycling and remanufacturing.

7.3 Future research directions

Further research may be conducted in the following areas:

1. The addition of warranty returns, commercial returns and production scraps to the model.
2. The consideration of other engine parts in the model.
3. The development of a strategic model capable of determining whether and where to open a new facility or expand the capacity of an existing facility (the proposed model takes into account the entire manufacturing and recycling process).
4. The consideration of revenue from selling rebuilt engines in the model.

5. The use of Visual Basic to make the model a decision support tool that can be easily applied by users (we used the commercial solver LINGO 9.0 with Microsoft Access as the database).

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APPENDICES

Appendix I Lists of Indices, parameters and decision variables

1. Indices

$a \in A$	Index for auto assembly plants
$c \in C$	Index for collection centers
$d \in D$	Index for dealerships
$i \in I$	Index for aluminum engine parts
$k \in K$	Index for transport modes
$m \in M$	Index for engine plants
$n \in N$	Index for engine types
$q \in Q$	Index for product families
$r \in R$	Index for recycling centers
$s \in S$	Index for aluminum casting plants
$t \in T$	Index for time periods
$u \in U$	Index for remanufacturing centers

2. Parameters

$AD1_{st}$:	Average demand for aluminum ingots at aluminum casting plant s in time period t (lbs/day)
$AD2_{st}$:	Average demand for recycled aluminum at aluminum casting plant s in time period t (lbs/day)
$AD3_{inst}$:	Average demand for engine part i for engine type n at aluminum casting plant s in time period t (units/day)
$AD4_{inmt}$:	Average demand for engine part i for engine type n at engine plant m in time period t (units/day)
$AD5_{nqmt}$:	Average demand for engine type n in product family q at engine plant m in time period t (units/day)
$AD6_{nqat}$:	Average demand for engine type n in product family q at auto assembly plant a in time period t (units/day)
$AD7_{nqat}$:	Average demand for vehicles with engine type n in product family q at auto assembly plant a in time period t (units/day)
$AD8_{ut}$:	Average demand for rebuilt engines at remanufacturing center u in time period t (units/day)
$AD9_{nqdt}$:	Number of vehicles with engine type n in product family q needed at the dealership d to sell to customers in time period t
$AD10_{dt}$:	Expected number of rebuilt engines needed at dealership d in time period t
BOM_{in}:	Bill of material utilization rate of engine part i per unit of engine type n
$CH1_{nqa}$:	Cost of handling a unit of vehicle with engine type n in product family q at auto assembly plant a
$CH2_{c}$:	Cost of handling a unit of end-of-life vehicle at collection center c
$CHLM_{mt}$:	Hiring cost per hour at engine plant m in time period t
$CLLM_{mt}$:	Layoff cost per hour at engine plant m in time period t

$COLM_{mt}$: Overtime labor cost per hour at engine plant *m* in time period *t*
$COLS_{st}$: Overtime labor cost per hour at aluminum casting plant *s* in time period *t*
$COLU_{ut}$: Overtime labor cost per hour at remanufacturing center *u* in time period *t*
$CP1$: Per unit weight purchasing cost of aluminum ingots from the suppliers
$CP2$: Per unit weight purchasing cost of recycled aluminum from recycling centers
$CP3_{miuk}$: The price of a unit of engine part *i*=3 shipped from engine plant *m* to remanufacturing center *u* using transport mode *k*
$CRLM_{mt}$: Regular labor cost per hour at engine plant *m* in time period *t*
$CRLS_{st}$: Regular labor cost per hour at aluminum casting plant *s* in time period *t*
$CRLU_{ut}$: Regular labor cost per hour at remanufacturing center *u* in time period *t*
$CTR1_{smk}$: Per unit cost of transportation from aluminum casting plant *s* to engine plant *m* using transport mode *k* ($/Full Truck Load, or $/FTL)
$CTR2_{mak}$: Per unit cost of transportation from engine plant *m* to auto assembly plant *a* using transport mode *k* ($/FTL)
$CTR3_{adk}$: Per unit cost of transportation from auto assembly plant *a* to dealership *d* using transport mode *k* ($/FTL)
$CTR4_{crk}$: Per unit cost of transportation from collection center *c* to recycling center *r* using transport mode *k* ($/FTL)
$CTR5_{muk}$: Per unit cost of transportation from engine plant *m* to remanufacturing center *u* using transport mode *k* ($/FTL)
$CTR6_{cuk}$: Per unit cost of transportation from collection center *c* to remanufacturing center *u* using transport mode *k* ($/FTL)
$CTR7_{udk}$: Per unit cost of transportation from remanufacturing center *u* to dealership *d* using transport mode *k* ($/FTL)
ELV_{ct}: Number of end-of-life vehicles collected at collection center *c* in time period *t* (equals to the expected number of vehicles retired over time multiplied by Ford's market share)
$FC1_{st}$: Storage space for holding aluminum ingots at aluminum casting plant *s* in time period *t* (cu.ft.)
$FC2_{st}$: Storage space for holding recycled aluminum at aluminum casting plant *s* at in time period *t* (cu.ft.)
$FC3_{inst}$: Storage space for holding engine part *i* for engine type *n* at aluminum casting plant *s* in time period *t* (cu.ft./unit)
$FC4_{inmt}$: Storage space for holding engine part *i* for engine type *n* at engine plant *m* in time period *t* (cu.ft./unit)
$FC5_{nqmt}$: Storage space for holding engine type *n* in product family *q* at engine plant *m* in time period *t* (cu.ft./unit)
$FC6_{nqat}$: Storage space for holding engine type *n* in product family *q* at auto assembly plant *a* in time period *t* (cu.ft./unit)
$FC7_{nqat}$: Storage space for holding vehicles with engine type *n* in product family *q* at auto assembly plant *a* in time period *t* (sq.ft./unit)
$FC8_{ut}$: Storage space for holding rebuildable engines at remanufacturing center *u* in time period *t* (cu.ft.)
$FC9_{ut}$: Storage space for holding rebuilt engines at remanufacturing center *u* in time period *t* (cu.ft.)

fm_m:	Ratio of overtime labor hours to regular time labor hours at the engine plant *m*
fs_s:	Ratio of overtime labor hours to regular time labor hours at the aluminum casting plant *s*
fu_u:	Ratio of overtime labor hours to regular time labor hours at the remanufacturing center *u*
$IVRA_a$:	Inventory carrying cost rate at the auto assembly plant *a*
$IVRC_c$:	Inventory carrying cost rate at collection center *c*
$IVRM_m$:	Inventory carrying cost rate at the engine plant *m*
$IVRS_s$:	Inventory carrying cost rate at the aluminum casting plant *s*
$IVRU_u$:	Inventory carrying cost rate at the remanufacturing center *u*
$LT1_{smk}$:	Transportation lead time from aluminum casting plant *s* to engine plant *m* using transport mode *k* (days)
$LT2_{mak}$:	Transportation lead time from engine plant *m* to auto assembly plant *a* using transport mode *k* (days)
$LT3_{adk}$:	Transportation lead time from auto assembly plant *a* to dealership *d* using transport mode *k* (days)
$LT4_{crk}$:	Transportation lead time from collection center *c* to recycling center *r* using transport mode *k* (days)
$LT5_{muk}$:	Transportation lead time from engine plant *m* to remanufacturing center *u* using transport mode *k* (days)
$LT6_{cuk}$:	Transportation lead time from collection center *c* to remanufacturing center *u* using transport mode *k* (days)
$LT7_{udk}$:	Transportation lead time from remanufacturing center *u* to dealership *d* using transport mode *k* (days)
$MLHM_{mt}$:	Maximum limit of labor hours allowed at the engine plant *m* in time period *t*
$MLHS_{st}$:	Maximum limit of labor hours allowed at the aluminum casting plant *s* in time period *t*
$MLHU_{ut}$:	Maximum limit of labor hours allowed at the remanufacturing center *u* in time period *t*
NN_{nqk}:	Number of engines of type *n* in product family *q* that can be loaded in one FTL of transport mode k
NNN_k:	Number of vehicles that can be loaded in one FTL of transport mode *k*
P1:	Percentage of aluminum ingots in the total amount of aluminum purchased by the aluminum casting plant *s*
P2:	Percentage of recycled aluminum in the total amount of aluminum purchased by the aluminum casting plant *s*
$PC1_{st}$:	Production capacity at aluminum casting plant *s* in time period *t*
$PC2_{mt}$:	Production capacity at engine plant *m* in time period *t*
$PC3_{at}$:	Production capacity at auto assembly plant *a* in time period *t*
$PC4_{ct}$:	Process capacity at collection center *c* in time period *t*
$PC5_{ut}$:	Production capacity at remanufacturing center *u* in time period *t*
PFH:	Per unit average price of a flattened hulk
PNE_{nq}:	Per unit price of the new engine type *n* in product family *q*
PNRE:	Per unit average price of rebuildable engines

PP_{in}:	Per unit price of the engine part *i* for engine type *n*
PRE:	Per unit average price of rebuilt engines
PV_{nq}:	The average price of a new vehicle with engine type *n* in product family *q*
RHM_{nqm}:	Per unit production time of engine type *n* in product family *q* at the engine plant *m* (in hrs.)
RHS_{ins}:	Per unit production time of engine part *i* for engine type *n* at the aluminum casting plant *s* (in hrs.)
RHU_u:	Per unit time to rebuild engines at the remanufacturing center *u* (in hrs.)
$STEA$:	Number of days to keep inventories of new engines at the auto assembly plants
$STEM$:	Number of days to keep inventories of new engines at the engine plants
$STEU$:	Number of days to keep inventories of rebuilt engines at the remanufacturing centers
$STPM$:	Number of days to keep inventories of engine parts at the engine plants
$STPS$:	Number of days to keep inventories of engine parts at the aluminum casting plant
$STRRS$:	Number of days to keep inventories of recycled aluminum at the aluminum casting plant
$STRS$:	Number of days to keep inventories of aluminum ingots at the aluminum casting plant
$STVA$:	Number of days to keep inventories of new vehicles at the auto assembly plants
VC_k:	The volume capacity of transport mode *k*
$VP1$:	The volume of a unit weight of aluminum ingot (cu.ft./lb)
$VP2$:	The volume of a unit weight of recycled aluminum (cu.ft./lb)
$VP3_{nq}$:	The volume of a unit of engine type *n* in product family *q* (cu.ft./unit)
$VP4_{in}$:	The volume of a unit of engine part *i* for engine type *n* (cu.ft./unit)
$VP5$:	The average amount of floor space a vehicle occupies (sq.ft./unit)
$VP6$:	The average volume of a flattened hulk (cu.ft./unit)
$VP7$:	The average volume of an engine (cu.ft./unit)
$VP8$:	The average volume of an engine part i=3 sold from engine plants (cu.ft/unit)
WC_k:	The weight capacity of transport mode *k*
$WP1$:	The average weight of engine part i=3
$WP2$:	The average weight of an engine
$WP3_{in}$:	The weight of one unit of engine part i=1 for engine type *n*
$WP4$:	The average weight of a flattened hulk
WRT:	The number of warranty replacements (units)

3. Decision variables

$SS1_{st}$:	Safety stock of aluminum ingots at aluminum casting plant *s* in time period *t*
$SS2_{st}$:	Safety stock of recycled aluminum at aluminum casting plant *s* in time period *t*
$SS3_{inst}$:	Safety stock of engine part *i* for engine type *n* at aluminum casting plant *s* in time period *t*

$SS4_{inmt}$:	Safety stock of engine part *i* for engine type *n* at engine plant *m* in time period *t*
$SS5_{nqmt}$:	Safety stock of engine type *n* in product family *q* at engine plant *m* in time period *t*
$SS6_{nqat}$:	Safety stock of engine type *n* in product family *q* at auto assembly plant *a* in time period *t*
$SS7_{nqat}$:	Safety stock of vehicles with engine type *n* in product family *q* at auto assembly plant *a* in time period *t*
$SS8_{ut}$:	Safety stock of rebuilt engines at remanufacturing center *u* at time period *t*
XC_{st}:	The amount of aluminum, in lbs, purchased by aluminum casting plant *s* in time period *t*
$XHLM_{mt}$:	Number of additional labor hours acquired at engine plant *m* in time period *t* through hiring
$XIT1_{sinmkt}$:	Number of units of engine part *i* for engine type *n* in transit between aluminum casting plant *s* and engine plant *m* using transport mode *k* in time period *t*
$XIT2_{mnqakt}$:	Number of units of engine type *n* in product family *q* in transit between engine plant *m* and auto assembly plant *a* using transport mode *k* in time period *t*
$XIT3_{anqdkt}$:	Number of units of vehicles with engine type *n* in product family *q* in transit between auto assembly plant *a* and dealership *d* using transport mode *k* in the time period *t*
$XIT4_{crkt}$:	Number of units flattened hulks in transit between collection center *c* and recycling center *r* using transport mode *k* in the time period *t*
$XIV1_{st}$:	Inventory of aluminum ingots, in lbs, held at the aluminum casting plant *s* at the end of time period *t*
$XIV2_{st}$:	Inventory of recycled aluminum, in lbs, held at the aluminum casting plant *s* at the end of time period *t*
$XIV3_{inst}$:	Number of units of engine part *i* for engine type *n* held as inventory at the aluminum casting plant *s* in time period *t*
$XIV4_{inmt}$:	Number of units of aluminum engine part *i* for engine type *n* held as inventory at the engine plant *m* at the end of time period *t*
$XIV5_{nqmt}$:	Number of units of engine type *n* in product family *q* held as inventory at the engine plant *m* at the end of time period *t*
$XIV6_{nqat}$:	Number of units of engine type *n* in product family *q* held as inventory at the auto assembly plant *a* at the end of time period *t*
$XIV7_{nqat}$:	Number of units of vehicles with engine type *n* in product family *q* held as inventory at the auto assembly plant *a* at the end of time period *t*
$XIV8_{ut}$:	Number of units of rebuildable engines held at the remanufacturing center *u* at the end of time period *t*
$XIV9_{ut}$:	Number of units of rebuilt engines held at the remanufacturing center *u* at the end of time period *t*
$XLLM_{mt}$:	Number of labor hours lost at engine plant *m* in time period *t* through layoffs
XM_{nqmt}:	Number of units of engine type *n* in product family *q* produced at engine

plant *m* in time period *t*

$XM1_{inst}$: Number of units of engine part *i* for engine type *n* produced at aluminum casting plant *s* in time period *t*

$XOLM_{mt}$: Number of overtime labor hours required at engine plant *m* in time period *t*

$XOLS_{st}$: Number of overtime labor hours required at aluminum casting plant *s* in time period *t*

$XOLU_{ut}$: Number of overtime labor hours required at remanufacturing center *u* in time period *t*

$XPS1_{sinmkt}$: Number of engine parts *i* for engine type *n* produced at aluminum casting plant *s* and sent to engine plant *m* using transport mode *k* in time period *t*

$XPS2_{mnqakt}$: Number of engine type *n* in product family *q* sent from engine plant *m* to auto assembly plant *a* using transport mode *k* in time period *t*

$XPS3_{anqdkt}$: Number of vehicles with engine type *n* in product family *q* sent from auto assembly plant *a* to dealership *d* using transport mode *k* in time period *t*

$XPS4_{crkt}$: Number of flattened hulks sent from collection center *c* to recycling center *r* using transport mode *k* in time period *t*

$XPS5_{miukt}$: Number of engine part *i*=3 shipped from engine plant *m* to remanufacturing center *u* using transport mode *k* in time period *t*

$XPS6_{cukt}$: Number of rebuildable engines processed at collection center *c* and sent to remanufacturing center *u* using transport mode *k* in time period *t*

$XPS7_{udkt}$: Number of rebuilt engines sent from remanufacturing center *u* to dealership *d* using transport mode *k* in time period *t*

$XRLM_{mt}$: Number of regular-time labor hours required at engine plant *m* in time period *t*

$XRLS_{st}$: Number of regular-time labor hours required at aluminum casting plant *s* in time period *t*

$XRLU_{ut}$: Number of regular-time labor hours required at remanufacturing center *u* in time period *t*

$XTR1_{smkt}$: Number of FTL shipments from aluminum casting plant *s* to engine plant m using transport mode *k* in time period *t*

$XTR2_{makt}$: Number of FTL shipments from engine plant *m* to auto assembly plant *a* using transport mode *k* in time period *t*

$XTR3_{adkt}$: Number of FTL shipments from auto assembly plant *a* to dealership *d* using transport mode *k* in time period *t*

$XTR4_{crkt}$: Number of FTL shipments from collection center *c* to recycling center *r* using transport mode *k* in time period *t*

$XTR5_{mukt}$: Number of FTL shipments from engine plant *m* to remanufacturing center *u* using transport mode *k* in time period *t*

$XTR6_{cukt}$: Number of FTL shipments from collection center *c* to remanufacturing center *u* using transport mode *k* in time period *t*

$XTR7_{udkt}$: Number of FTL shipments from remanufacturing center *u* to dealership *d* using transport mode *k* in time period *t*

$XUS1_{st}$: Amount of aluminum used at aluminum casting plant *s* in time period *t*

$XUS2_{inmt}$: Number of units of engine part *i* for engine type *n* used at the engine plant *m* in time period *t*

$XUS3_{nqat}$: Number of units of engine type n in product family q used in the production of vehicles at auto assembly plant a in time period t

$XUS4_{ut}$: Number of rebuildable engines processed at remanufacturing center u in time period t

Appendix II (CD Format) LINGO script

Appendix III (CD Format) Complete results

The CD attached at the end of this thesis provides the appendices

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